Figures

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Figure 1. The Cartesian coordinate aligned with the uniform gravitational acceleration (which is 2.74×10^4 cm sec^{-2}) will be called the *altitude* in the plots that follow and will be measured in Mm upward from the bottom of the simulation. Four different average mass fluxes are plotted in this figure. The blue squares, which run pretty flat along the zero line are $\langle \rho u_{\parallel} \rangle$; which, for all intents and purposes, is zero. The red squares (which are a bit hard to discern from the red circles) are $|\langle \rho \mathbf{u}_{\perp} \rangle|$. This is a positive-definite quantity. The direction $\langle \rho \mathbf{u}_{\perp} \rangle$ points does change (overall by about 90 degrees) with altitude, but we have not bothered to plot this. Statistically speaking, no horizontal direction should be preferred over any other (the boundary conditions are periodic). If the simulation were run for longer periods of time, one would expect $\langle \rho \mathbf{u}_{\perp} \rangle$ to tend to zero. The blue circles are the product $\langle \rho \rangle \langle u_{\parallel} \rangle$, with the convention that a positive velocity is one that moves fluid to a higher altitude. Because the mean density stratificaiton is positive-definite (see Figure 2), this implies that the (approximately) lower half of the simulation has a sustained average upflow, while the upper half has a much smaller downflow concentrated near its base. The vertical line in this, and subsequent plots, marks the convergence point for these two average flows. The red circles are $\langle \rho \rangle |\langle \mathbf{u}_{\perp} \rangle| \approx |\langle \rho \mathbf{u}_{\perp} \rangle|$.

Figure 2. This is the mean stratification for the density, $\langle \rho \rangle$. The vertical line marks the location of the convergence point for the mean upflow/downflow transition shown in Figure 1.

Figure 3. Here we remove the density contrast and plot $\langle u_{\parallel} \rangle$ in blue circles, and $|\langle \mathbf{u}_{\perp} \rangle|$ in red circles. Notice that there is a upflow/downflow divergence point near the very top of the simulation which was masked by the factor of density used in Figure 1. MURaM tries to limit the influence of the upper boundary on the simulation by allowing things to seamlessly (as much as possible) pass through this boundary if that is their intent. Details of just what this means can be found in the MURaM reference. For this reason, the mean velocity develops an outflow, which, although it is in fact large compared to the mean elsewhere in the simulation, has very little consequence since, from Figure 2, there is essentially no material here.

Figure 4. This is the mean stratification for the gas pressure, $\langle p \rangle$. The vertical line marks the location of the convergence point for the mean upflow/downflow transition shown in Figure 1.

Figure 5.(*a*) In blue circles we plot $\langle |\mathbf{u}|^2 \rangle^{1/2}$ and in red circles $|\langle \mathbf{u} \rangle|$. We've already seen the various contributions to the red circles in Figure 3. This plot indicates that random or turbulent motions of the fluid are much larger in size than the mean flows. (*b*) Here we plot separately the various components that contribute to $\langle |\mathbf{u}|^2 \rangle^{1/2}$. The blue circles provide the contribution from the

vertical turbulent motions, while the two components of the horizontal turbulent motions are plotted separately with red circles. Notice they tend to lie on top of one another, consistent with the fact that the simulation has no particular preference for either of the two horizontal directions. With the green circles we have just summed the two red curves. The turbulence is said to be *isotropic* if all three components of the velocity contribute equally. This obtains between altitudes of 3.2 and 3.8 Mm and precisely at our upflow/downflow convergence point!

Figure 6. This is the mean stratification for the temperature, $\langle T\rangle$. The vertical line marks the location of the convergence point for the mean upflow/downflow transition shown in Figure 1. The lower (approximate) half of the simulation has a constant mean temperature gradient, while the upper half is nearly isothermal.

Figure 7. This figure is similar in format to Figure 5(a), but for the magnetic field instead of the velocity field. The red circles are $|\langle \mathbf{B}_\perp\rangle| \approx |\langle \mathbf{B}\rangle|$. The comparable plot for $\langle B_\parallel\rangle$ lies below the bottom of this Figure indicating that there is essentially no mean vertical flux at any time. The blue circles are $\langle|\mathbf{B}|^2\rangle^{1/2}$. The discrepency in size is a consequence of the fact that the simulation again finds it difficult to pick a horizontal direction in which to build a mean flux. The direction of $\langle \mathbf{B}_\perp\rangle$, which is not plotted here, is all over the place. This plot is characteristic of a *local* magnetic dynamo—it makes a lot of magnetic field but hardly any magnetic flux. If the simulation were rotating about some direction not parallel to gravity, then an important horizontal direction is picked out (north, say!) and the dynamo now has the opportunity to generate a horizontal flux. Because $\nabla\cdot\mathbf{B} = 0$, whatever vertical flux is present initially is maintained for all subsequent times. In this case, zero.

Figure 8. This plot answers the question how important, dynamically, is the magnetic field that is generated in this simulation. The red circles are $\langle|\mathbf{B}/\sqrt{p}|^2\rangle$ times $1/8\pi$. The horizontal line indicates the location where the gas pressure and the magnetic pressure are comparable in size. Therefore we expect the Lorentz Force to be important dynamically everywhere in the upper third of the simulation and of minor consequence (except perhaps in some isolated locations) in the lower half.

Figure 9. This plot is similar in format to Figure 5(b) except now for the dimensionless quantity plotted in Figure 8. Notice that the contribution of the vertical magnetic field is always more imporant than either of the horizontal components individually. This is consistent with magnetic fields being buoyant and wanting to align themselves with gravity—an effect that is most pronounced in the vicinity of the upflow/downflow convergence zone.

Figure 10. This is a plot of the mean Planck Function $\sigma_R\langle T^4\rangle/\pi$. On average, an energy flux of 6.3×10^{10} erg cm^{-2} sec^{-1} is continutally forced through the computational domain. This value is indicated by the horizontal line. Notice that this too matches up well with the value of the mean Planck Function in the vicinity of the upflow/downflow convergence zone.

Figure 11. This is the same mean temperature $\langle T \rangle$ plotted in Figure 6, but now plotted against $\langle \log \tau \rangle$ computed at each altitude in Figure 6. Here, τ is the vertical optical depth measured into the computational domain starting at ≈ 0 at the upper boundary and increasing inward. The upflow/downflow convergence point is very near the average optical depth unity surface.

Figure 12. As in Figure 11, the blue circles are again the mean temperature $\langle T \rangle$ plotted plotted against $\langle \log \tau \rangle$. The red circles are $\langle T^2 \rangle^{1/2}$ and the green circles are $\langle T^4 \rangle^{1/4}$. This indicates that the Planck Function, at least above an optical depth of 10^{-5}, is *much* larger than $\sigma_R \langle T \rangle^4 / \pi$ would suggest. In this region, intermittent pulses of very high temperature material begin to dominate the mean of the Planck Function, but have less impact on the average temperature.

Figure 13. Purely for your amusement, Figures (a) through (e) show the mean of the Planck Function (i.e., Figure 10) plotted against a restricted mean optical depth scale $\langle \tau \rangle$. Like we found for T it is definitely the case at small optical depths that $\log\langle \tau \rangle \neq \langle \log \tau \rangle$—to be clear, in these plot, the abscissa is actually $\langle \tau \rangle$. The dependence of the mean Planck Function, which is the mean source function for MURaM because it assumes LTE everywhere, does not follow any sensible $a + b\langle \tau \rangle$ behavior that we might expect from radiative effects alone. We are seeing very clearly the divergence of a complicated mechanical energy flux carried by the convection and magnetic fields that is contributing to the energy budget of the material.