Astrophysical Radiation Magnetohydrodynamics

Being an Opera in Four Acts

ACTS III & IV

Lecture 2: Applications

SHOCKS & *Supernovae*

With Words & Music By

Tom Bogdan

(aka Rick Blaine)

Paul Charbonneau

(aka Captain Louis Renault)

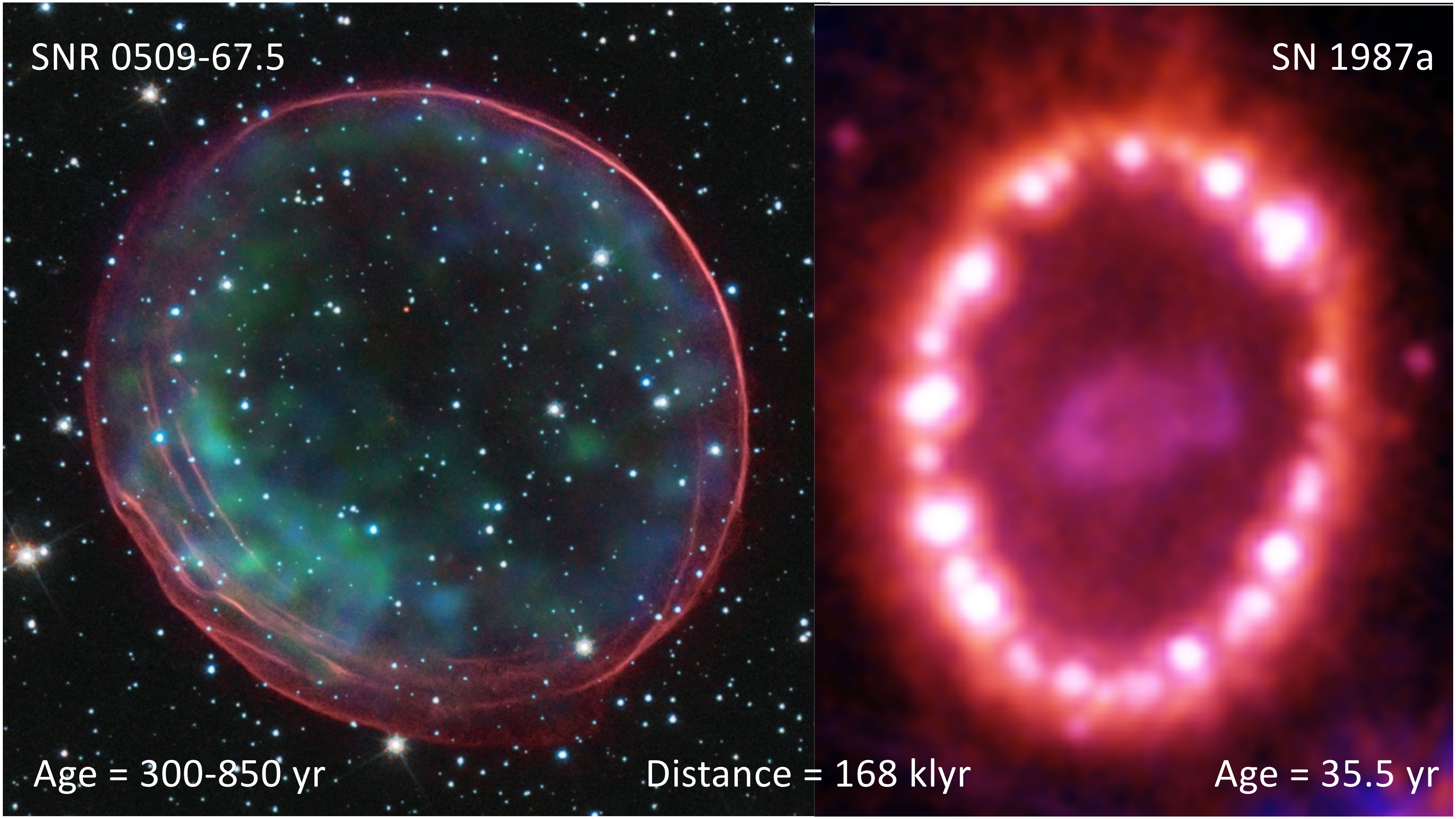
25 September 2022

Rick: How can you close me up? On what grounds?

Renault: I'm **shocked**, shocked to find gambling going on!

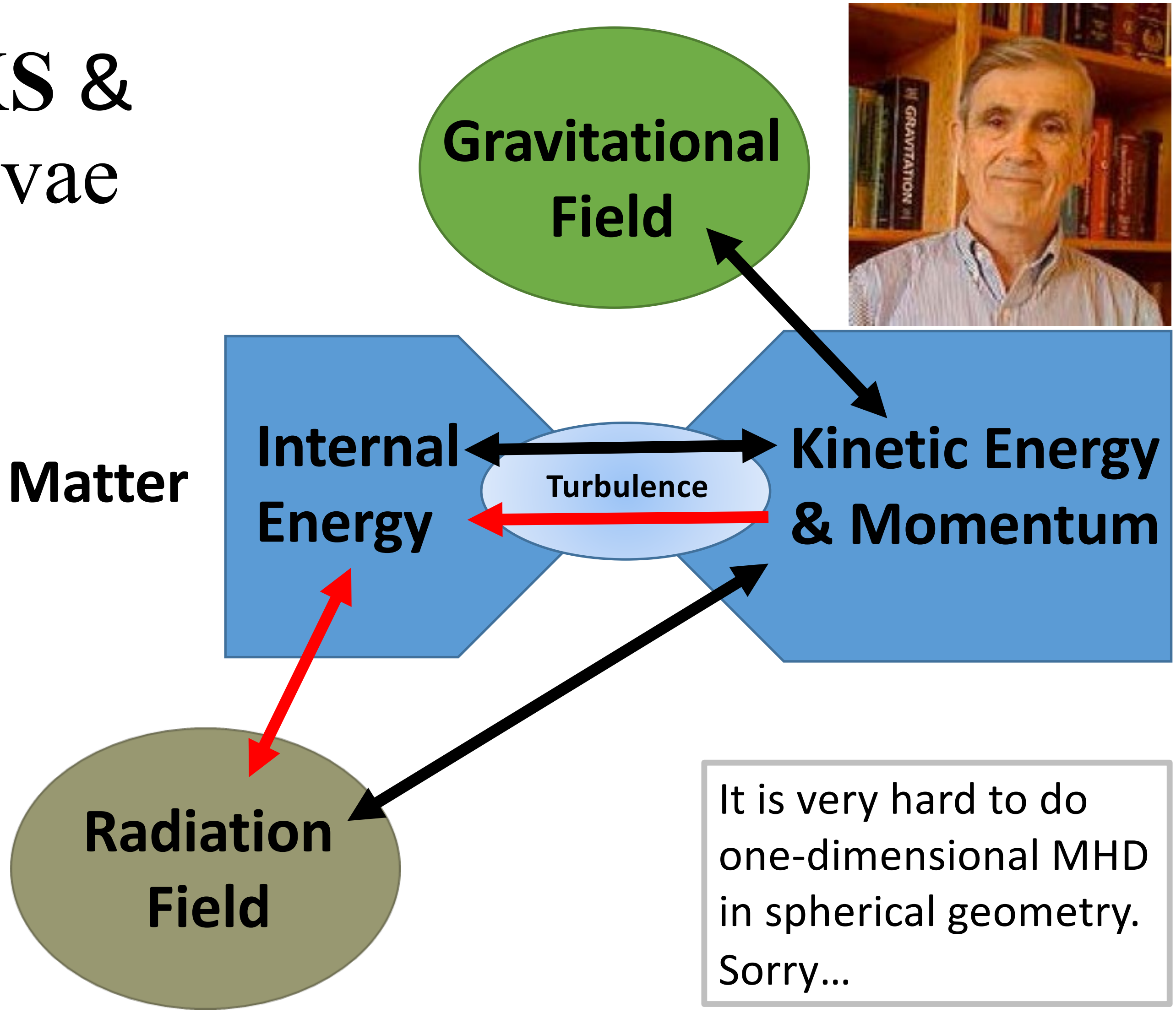
Croupier: Your winnings, sir.

Renault : [sotto voce] Oh, thank you very much. [aloud] Everyone


SNR 0509-67.5
SN 1987a
Age = 300-850 yr
Distance = 168 klyr
Age = 35.5 yr

SHOCKS & Supernovae

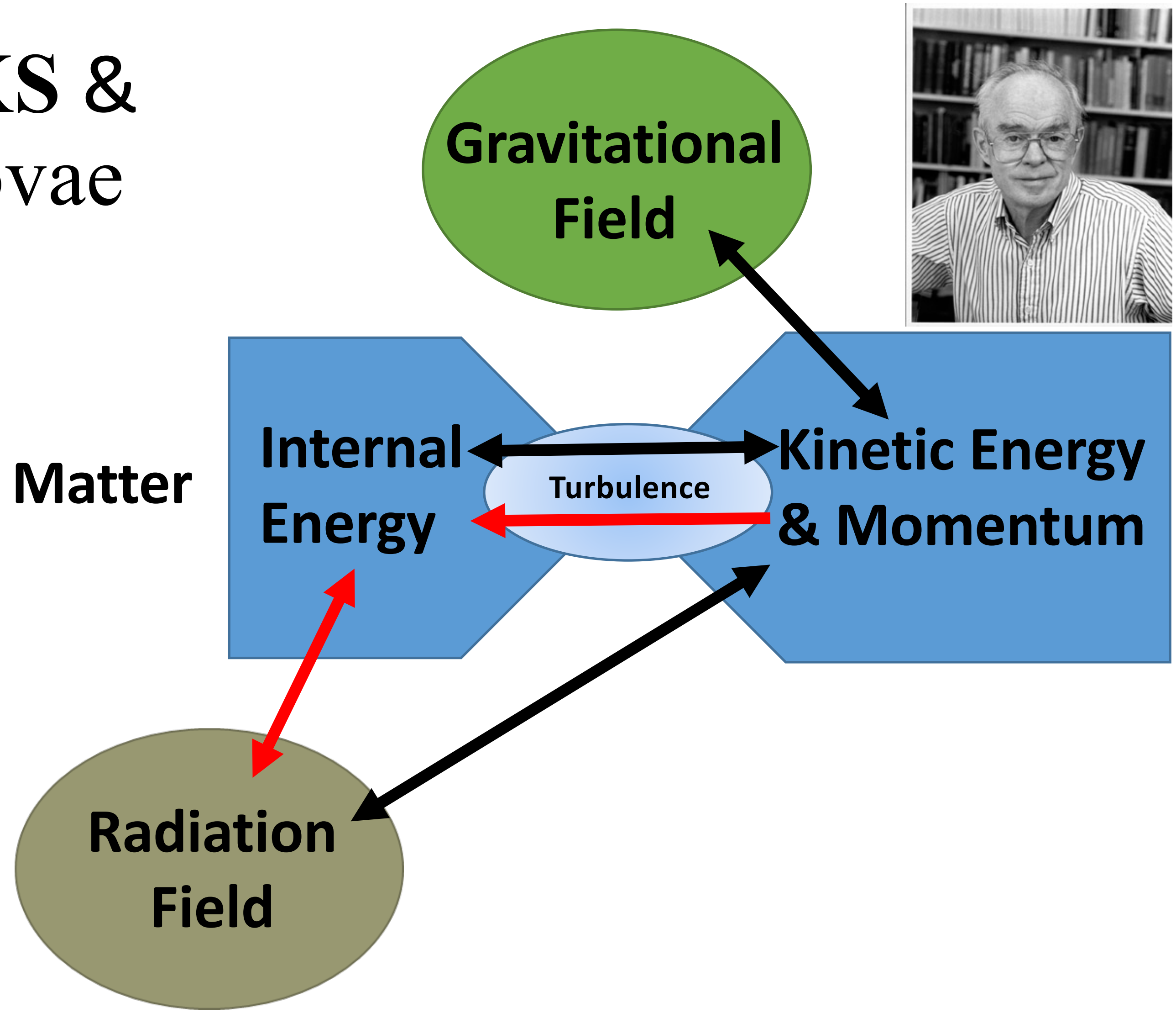
"An essential difference between the dynamics of radiating and non-radiating fluids is that because photons typically have much longer mean free paths than their material counterparts, they can introduce a fundamental global coupling between widely separated parts of the flow, which must be treated by a full transport theory."
--**Dimitri Mihalas**



It is very hard to do one-dimensional MHD in spherical geometry. Sorry…

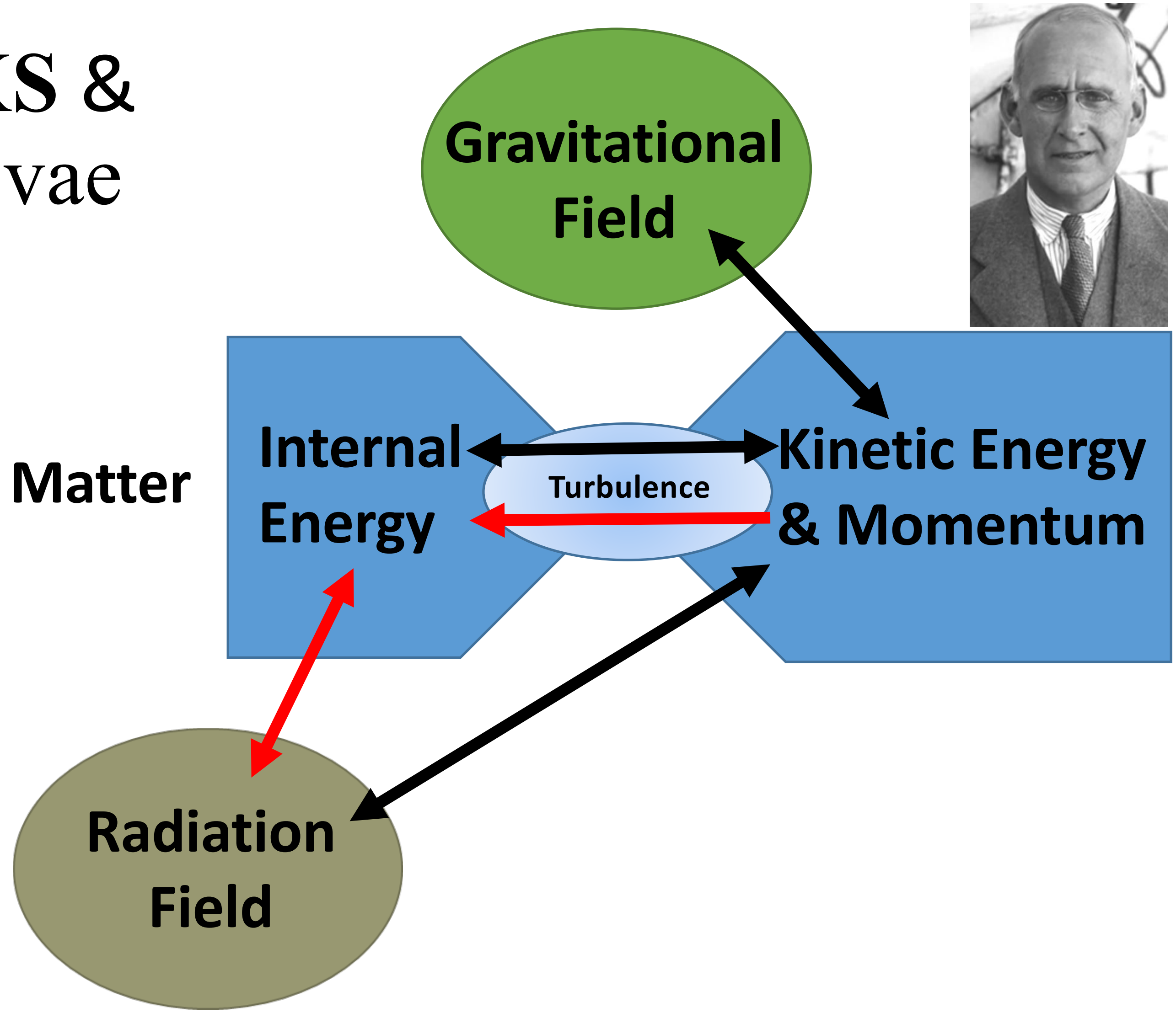
SHOCKS & Supernovae

"I've always enjoyed learning how things work. ... Simply endless puzzles and problems that come to light, some of them trivial, amusing, some of them very important and I take great pleasure in learning them."
--**Eugene Parker**



SHOCKS & Supernovae

“We do not argue with the critic who urges that the stars are not hot enough for this process; we tell him to go and find a hotter place.”
--**Arthur Eddington**

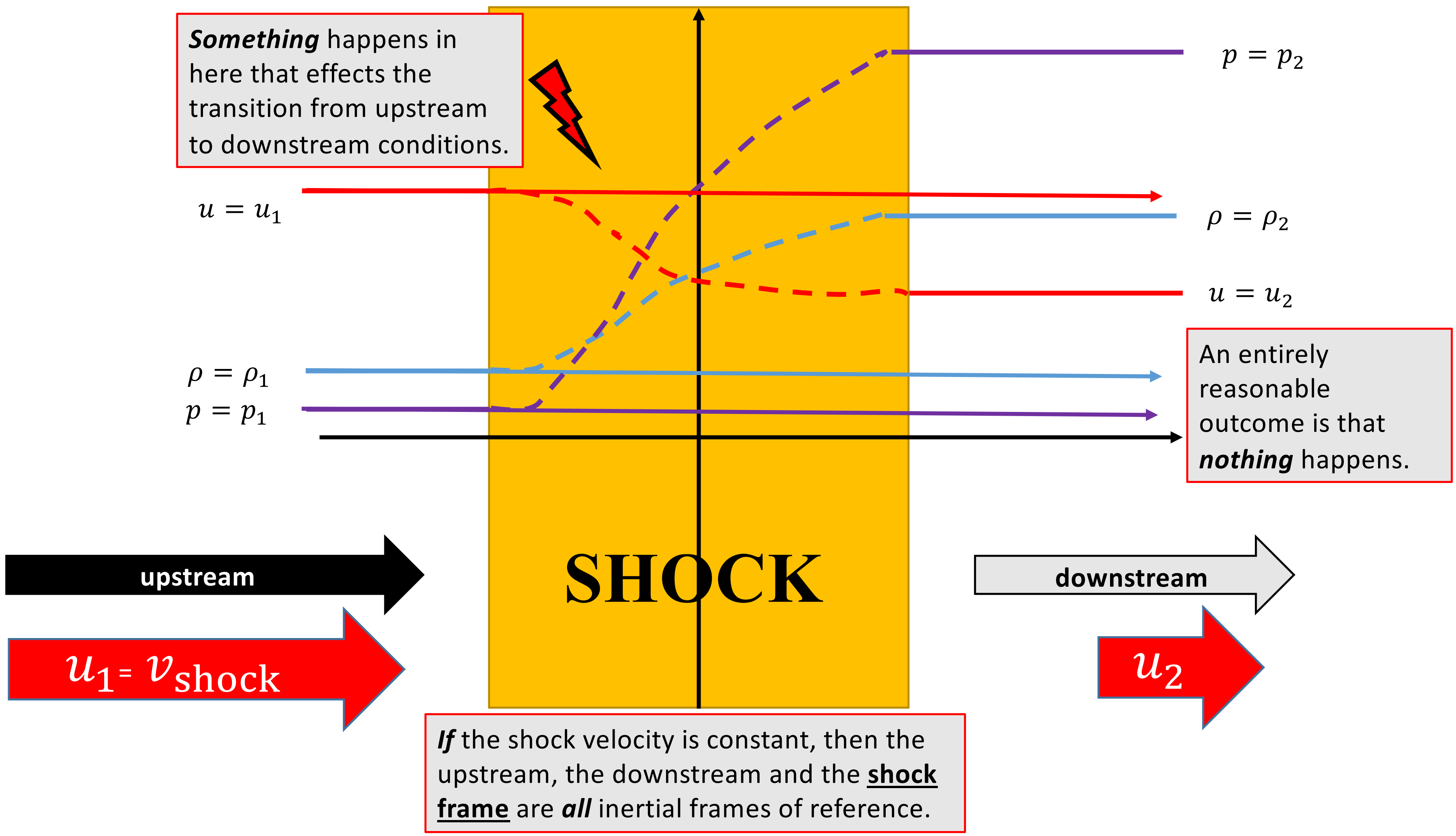


Astrophysical Radiation Magnetohydrodynamics

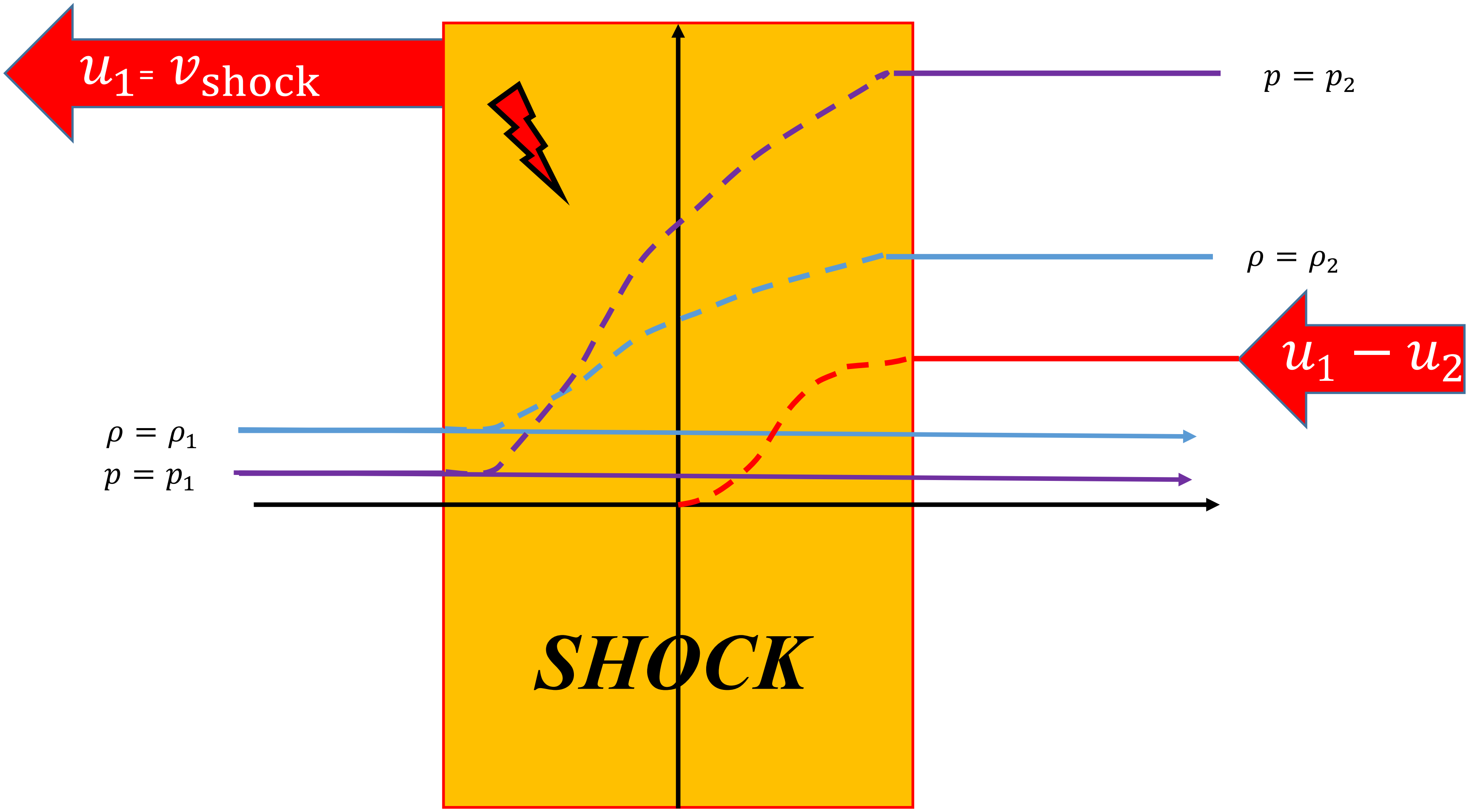
Being an Opera in Four Acts

ACT III

We at some times are minions of our theories,
The fault, dear Brutus, is not in ourselves,
But in our stars, that we are underlings.



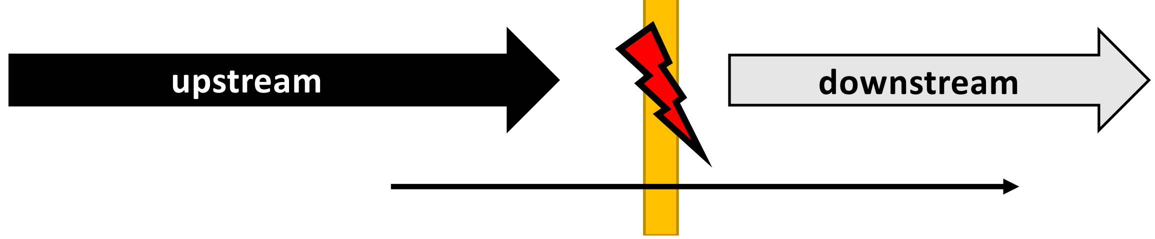
Something happens in here that effects the transition from upstream to downstream conditions.
$u = u_1$
$\rho = \rho_1$
$p = p_1$
$p = p_2$
$\rho = \rho_2$
$u = u_2$
An entirely reasonable outcome is that ***nothing*** happens.
SHOCK
upstream
downstream
$u_1 = v_{shock}$
u_2
If the shock velocity is constant, then the upstream, the downstream and the **shock frame** are ***all*** inertial frames of reference.



If the shock velocity is constant, then the **upstream**, the downstream and the shock **frame** are ***all*** inertial frames of reference.

SHOCK

Mass Conservation	$$\frac{\partial}{\partial t}\rho + \boldsymbol{\nabla}\cdot\rho\boldsymbol{u} = 0$$
Momentum Conservation	$$\frac{\partial\mathfrak{P}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbb{\Pi} = -\rho\,\boldsymbol{\nabla}\Phi$$
Energy Conservation	$$\frac{\partial\mathfrak{E}}{\partial t} + \boldsymbol{\nabla}\cdot\mathfrak{F} = 0$$



❶ $$\mathcal{M} = \rho u$$ ❶❷

❷ $$\mathcal{P} = \rho u^2 + p + P - \tau$$ ❸ ❻

❸ $$\mathcal{L} = \mathcal{M}\left(\frac{1}{2}u^2 + \frac{\gamma}{\gamma-1}\frac{p}{\rho}\right) + \left(F - \frac{\alpha}{\nu+1}\frac{\mathrm{d}}{\mathrm{dz}}T^{\nu+1}\right)$$ ❺

❹ $$\frac{p}{\rho} = \frac{\mathfrak{R}}{\mu}T$$ ❹

Les ondes de choc (1.79)

Les ondes de choc (1.80)

Les ondes de choc (1.81)

You have seen this all before!

…but this part, maybe not!

$$\boldsymbol{F} = \boldsymbol{F}' + \boldsymbol{u}[E' + \mathbb{P}'] + \cdots$$

$$\mathbb{P} = \mathbb{P}' + \frac{1}{c^2}[\boldsymbol{u}\boldsymbol{F}' + \boldsymbol{F}'\boldsymbol{u}] + \cdots$$

$$\zeta = \frac{P_1}{p_1}$$

Your HOMEWORK Assignment!

Include $\mathbb{M}$.

$$\frac{1}{\eta_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$$

$$\Pi_2 = \frac{p_2}{p_1} = \frac{2\gamma M^2 - (\gamma-1)}{(\gamma+1)}$$

$$s_1 < s_2$$

$$M^2 = \frac{\rho_1 u_1^2}{\gamma p_1}$$

Les ondes de choc (1.82)

Les ondes de choc (1.85)

If the upstream gas is ice cold...

$$\gamma M^2 p_1 = \rho_1 u_1^2$$

$$\gamma M^2 \to \infty, \quad p_1 \to 0$$

$$p_2 = \frac{2}{\gamma+1}\rho_1 u_1^2 - \frac{\gamma-1}{\gamma+1} p_1$$

$$\frac{1}{\eta_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$$

$$\Pi_2 = \frac{p_2}{p_1} = \frac{2\gamma M^2 - (\gamma-1)}{(\gamma+1)}$$

$$s_1 < s_2$$

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Les ondes de choc (1.82)

Les ondes de choc (1.85)

$$T_2 = T_1$$

If the shock is optically-thin…

$$\frac{1}{\eta_2} = \frac{\rho_2}{\rho_1} = \Pi_2 = \frac{p_2}{p_1} = \gamma M^2 = \frac{\rho_1 u_1^2}{p_1}$$

Check the factor of gamma

$\gamma = 1$

$$\frac{1}{\eta_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$$

$$\Pi_2 = \frac{p_2}{p_1} = \frac{2\gamma M^2 - (\gamma-1)}{(\gamma+1)}$$

$$s_1 < s_2$$

$$M^2 = \frac{\rho_1 u_1^2}{\gamma p_1}$$

Les ondes de choc (1.82)

Les ondes de choc (1.85)

1 $\mathcal{M} = \rho u$

2 $\mathcal{P} = \rho u^2 + p + P - \tau$

3 $\mathcal{L} = \mathcal{M}\left(\frac{1}{2}u^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho}\right) + \left(F - \frac{\alpha}{\nu + 1}\frac{\mathrm{d}}{\mathrm{d}z}T^{\nu+1}\right)$

4 $\frac{p}{\rho} = \frac{\Re}{\mu}T$

Les ondes de choc (1.79)

Les ondes de choc (1.80)

Les ondes de choc (1.81)

$$\boldsymbol{F} = \boldsymbol{F}' + \boldsymbol{u}[E' + \mathbb{P}'] + \cdots$$

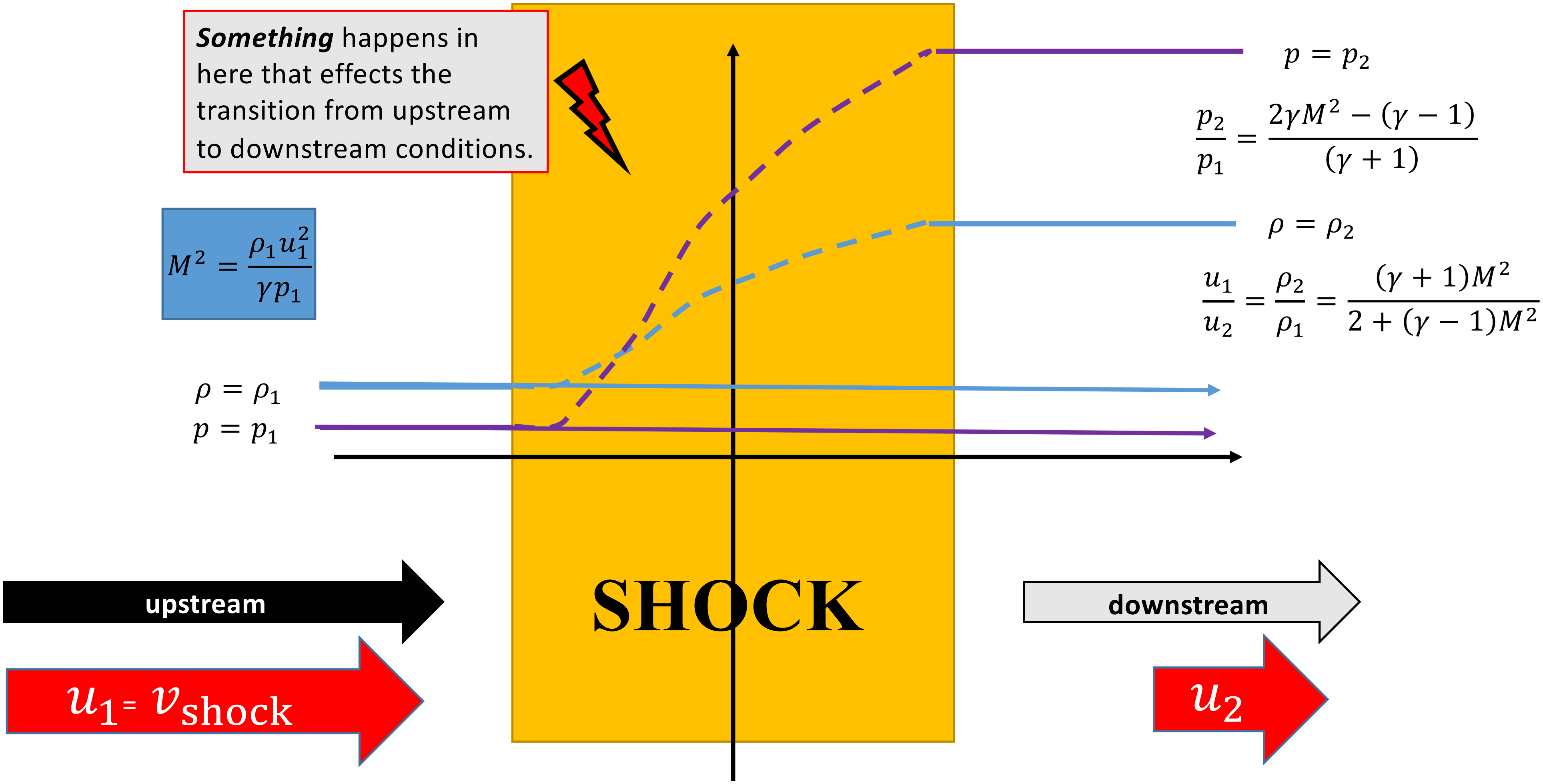
$$\mathbb{P} = \mathbb{P}' + \frac{1}{c^2}[\boldsymbol{u}\boldsymbol{F}' + \boldsymbol{F}'\boldsymbol{u}] + \cdots$$

$$\zeta = \frac{P_1}{p_1}$$

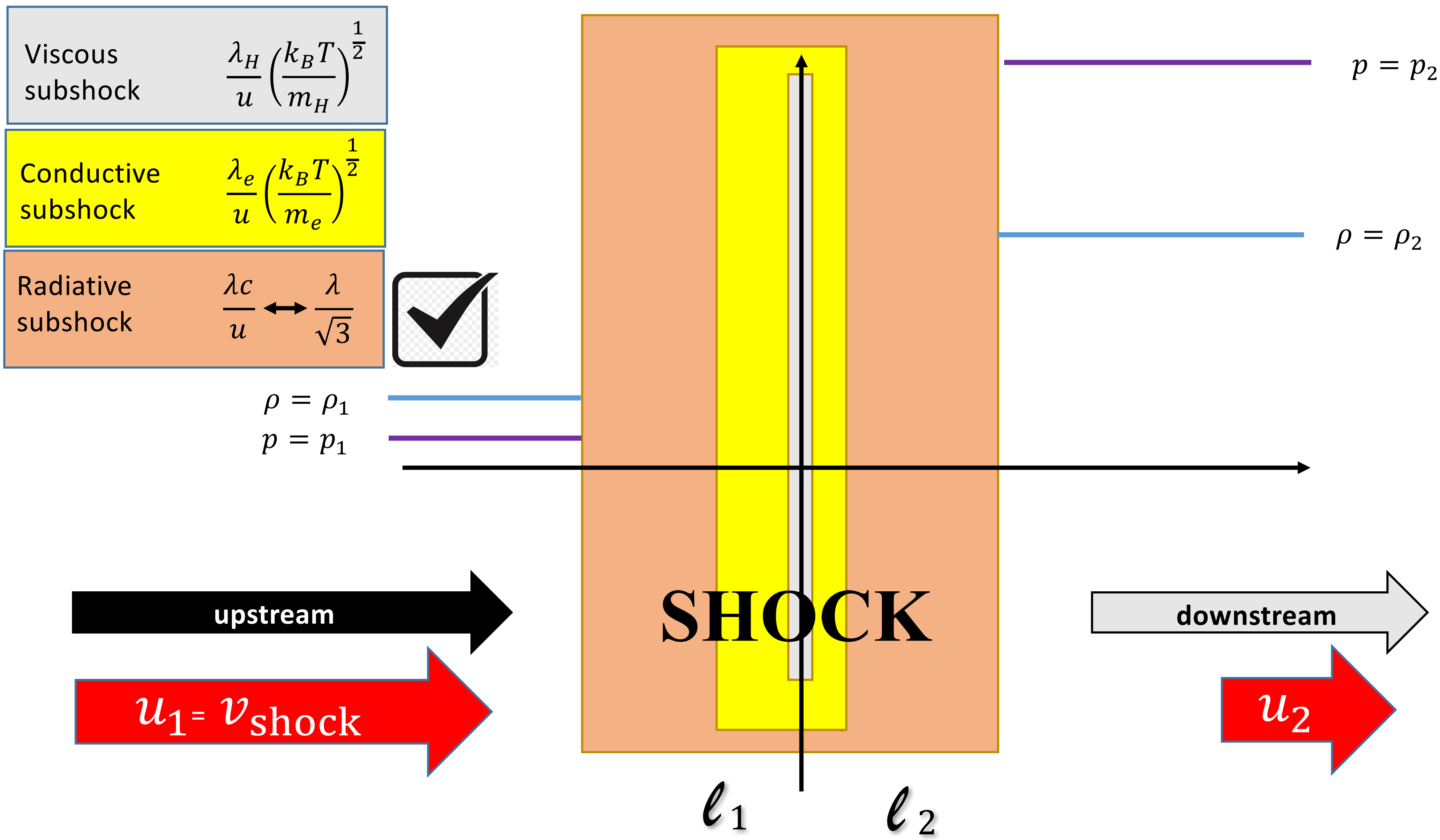
$$M^2 = \frac{\rho_1 u_1^2}{\gamma p_1}$$

Shocks,
subshocks,
subsubshocks…

$s_1 < s_2$



Something happens in here that effects the transition from upstream to downstream conditions.
$M^2 = \frac{\rho_1 u_1^2}{\gamma p_1}$
$p = p_2$
$\frac{p_2}{p_1} = \frac{2\gamma M^2 - (\gamma - 1)}{(\gamma + 1)}$
$\rho = \rho_2$
$\frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M^2}{2 + (\gamma - 1)M^2}$
$\rho = \rho_1$
$p = p_1$
SHOCK
upstream
downstream
$u_1 = v_{shock}$
u_2



Viscous subshock $\frac{\lambda_H}{u}\left(\frac{k_B T}{m_H}\right)^{\frac{1}{2}}$
Conductive subshock $\frac{\lambda_e}{u}\left(\frac{k_B T}{m_e}\right)^{\frac{1}{2}}$
Radiative subshock $\frac{\lambda c}{u} \leftrightarrow \frac{\lambda}{\sqrt{3}}$
$\rho = \rho_1$
$p = p_1$
$p = p_2$
$\rho = \rho_2$
SHOCK
ℓ_1
ℓ_2
upstream
$u_1 = v_{\mathrm{shock}}$
downstream
u_2

1 $\mathcal{M} = \rho u$ (1)(2)

2 $\mathcal{P} = \rho u^2 + p + P$ (3)(4)

3 $\mathcal{L} = \mathcal{M}\left(\frac{1}{2}u^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho}\right) + F$ (5)

$$\zeta = \frac{P_1}{p_1}$$

$$M^2 = \frac{\rho_1 u_1^2}{\gamma p_1}$$

$$\mathrm{Bo} = \frac{\rho u c_P T}{\sigma_R T^4}$$

≈ 1 for the solar surface
≈ 10^{-4} for an O star surface

$$\boldsymbol{F} = \boldsymbol{F}' + \boldsymbol{u}[E' + \mathbb{P}'] + \quad \rightarrow \quad R = \frac{\rho e}{E} = \frac{1}{2}\frac{p}{P} = \frac{3k_B}{2\mu a_R m_H}\frac{\rho}{T^3}$$

≈ 10,000 for the solar surface
≈ 0.1 for an O star surface

$$\mathbb{P} = \mathbb{P}' + \frac{1}{c^2}[\boldsymbol{u}\boldsymbol{F}' + \boldsymbol{F}'\boldsymbol{u}] \; ;$$

at most u/c times P

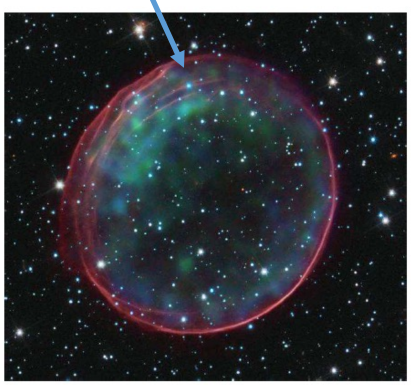


Figure 1.5: Onde de détonation produite par une supernova ayant pêté il y a plus de 400 ans (SNR 0509-67.5). Cette imagine combine des données optiques (rouge) et en rayon-X (vert-bleu). L'émission optique est associée au chauffage du milieu interstellaire par le passage de l'onde de choc. Image de la NASA, en domaine public.

Unless we are in a very hot and tenuous environment, like the envelope of an O star, all the terms in yellow can be safely dropped.

Otherwise…

① $\mathcal{M} = \rho u$ ❶❷

② $\mathcal{P} = \rho u^2 + p + P$ ❸ ❹

③ $\mathcal{L} = \mathcal{M}\left(\frac{1}{2}u^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho}\right) + F$ ❺

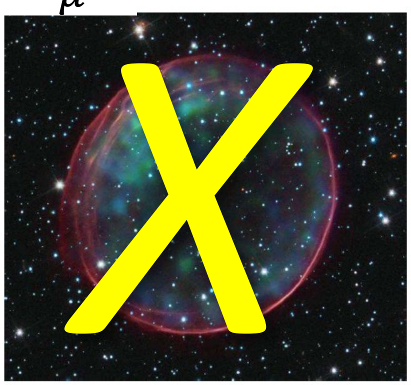
④ $\frac{p}{\rho} = \frac{\Re}{\mu}T$ ❻

⑤ $E' = 3P' = \frac{4\sigma_R}{c}T^4$

LTE dynamic diffusion limit!

Otherwise…

⑥ $\boldsymbol{F} = \mathbf{0} + \boldsymbol{u}[E' + \mathbb{P}'] + \cdots$



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As $M \longrightarrow \infty$, $\eta_2 \longrightarrow 1/7$ if $\zeta \neq 0$
Π_2 and T_2/T_1 are depressed

Your HOMEWORK Assignment!

$$\eta_2 = \frac{\rho_1}{\rho_2} = ? \qquad M^2 = \frac{\rho_1 u_1^2}{\gamma p_1} \qquad \zeta = \frac{P_1}{p_1}$$

$$\Pi_2 = \frac{p_2}{p_1} = ?$$

Hint: For given ζ and η, eliminate the u^2 terms between ② and ③ to find a 5th order polynomial for Π. Solve. Then determine M^2. Set $\zeta = 0$ to recover (1.82) and (1.85).

1 $\mathcal{M} = \rho u$ (1)(2)

2 $\mathcal{P} = \rho u^2 + p$ (3)

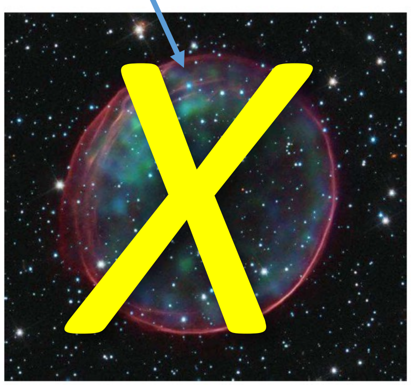
3 $\mathcal{L} = \mathcal{M}\left(\frac{1}{2}u^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho}\right) + F$ (5)

4 $E = 3P$ (6) (4)

$$M^2 = \frac{\rho_1 u_1^2}{\gamma p_1}$$

$\boldsymbol{F} = \boldsymbol{F}'$

$\mathbb{P} = \mathbb{P}'$



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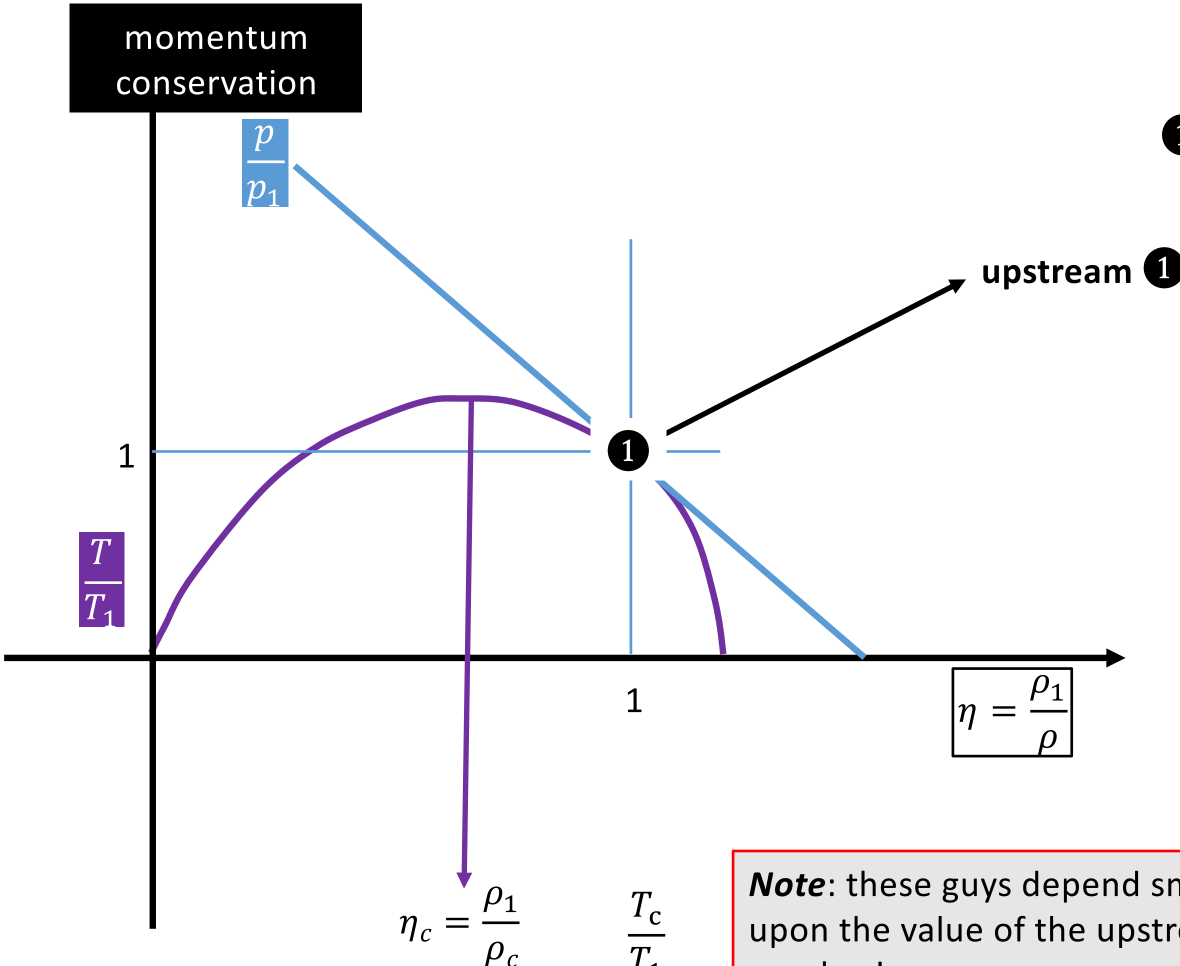
(7)

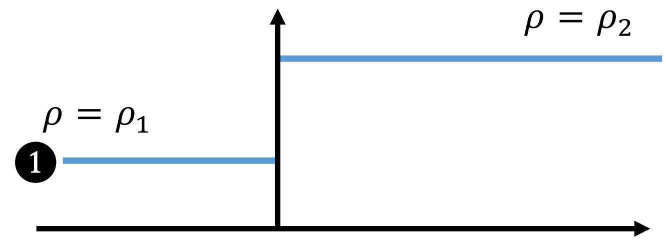
5 $\nabla \cdot \boldsymbol{F} = \kappa\left[4\sigma_R T^4 - cE\right]$

6 $\nabla \cdot \mathbb{P} = -\frac{\kappa}{c}\boldsymbol{F}$

7 $\frac{p}{\rho} = \frac{\Re}{\mu} T$

Optically-thick, non-LTE,
radiation pressure neglected, limit.

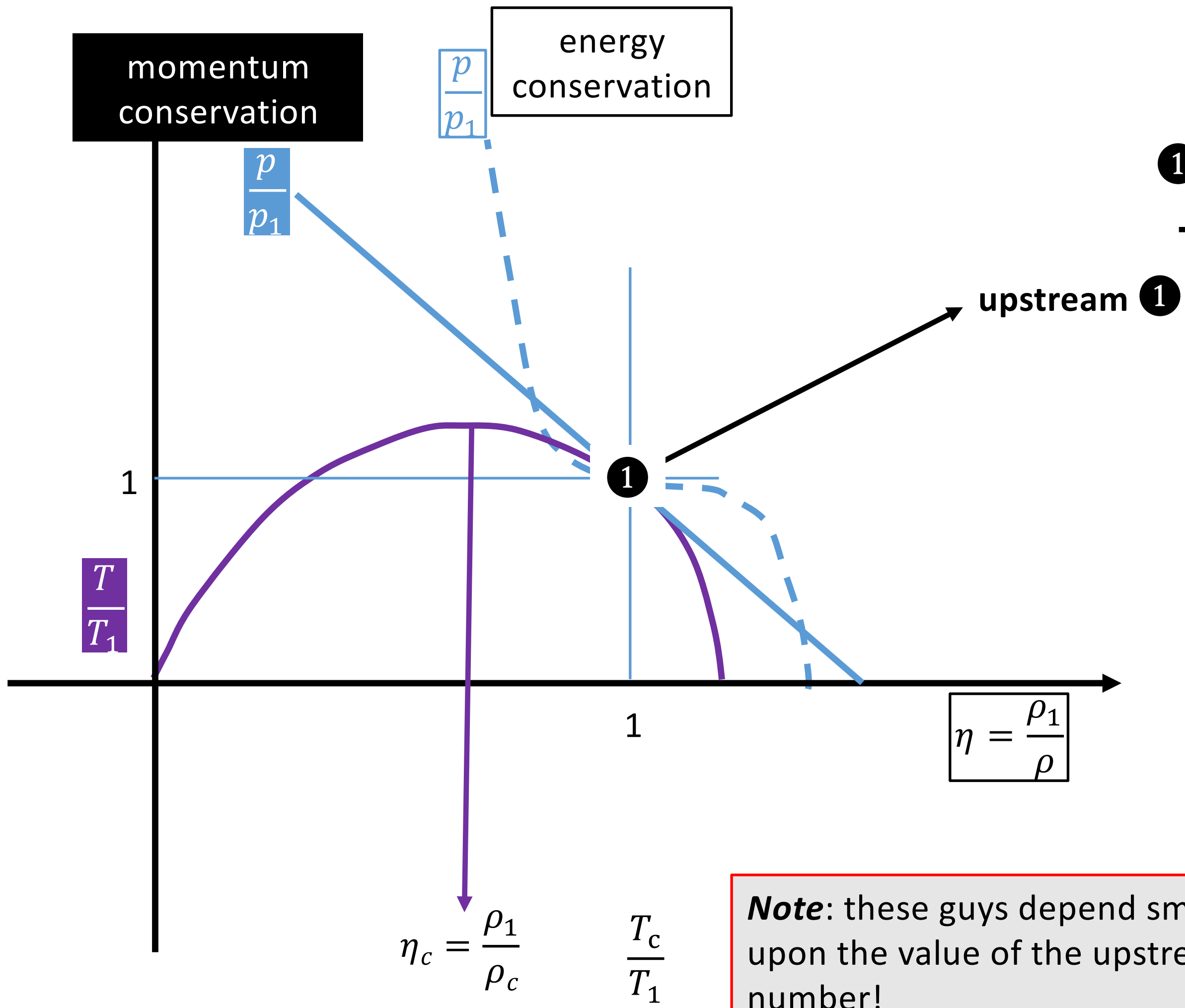


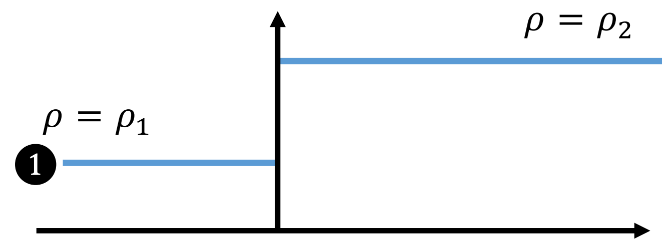


For a ***given***

$$M^2 = \frac{\rho_1 u_1^2}{\gamma p_1}$$

Note: these guys depend smoothly upon the value of the upstream Mach number!





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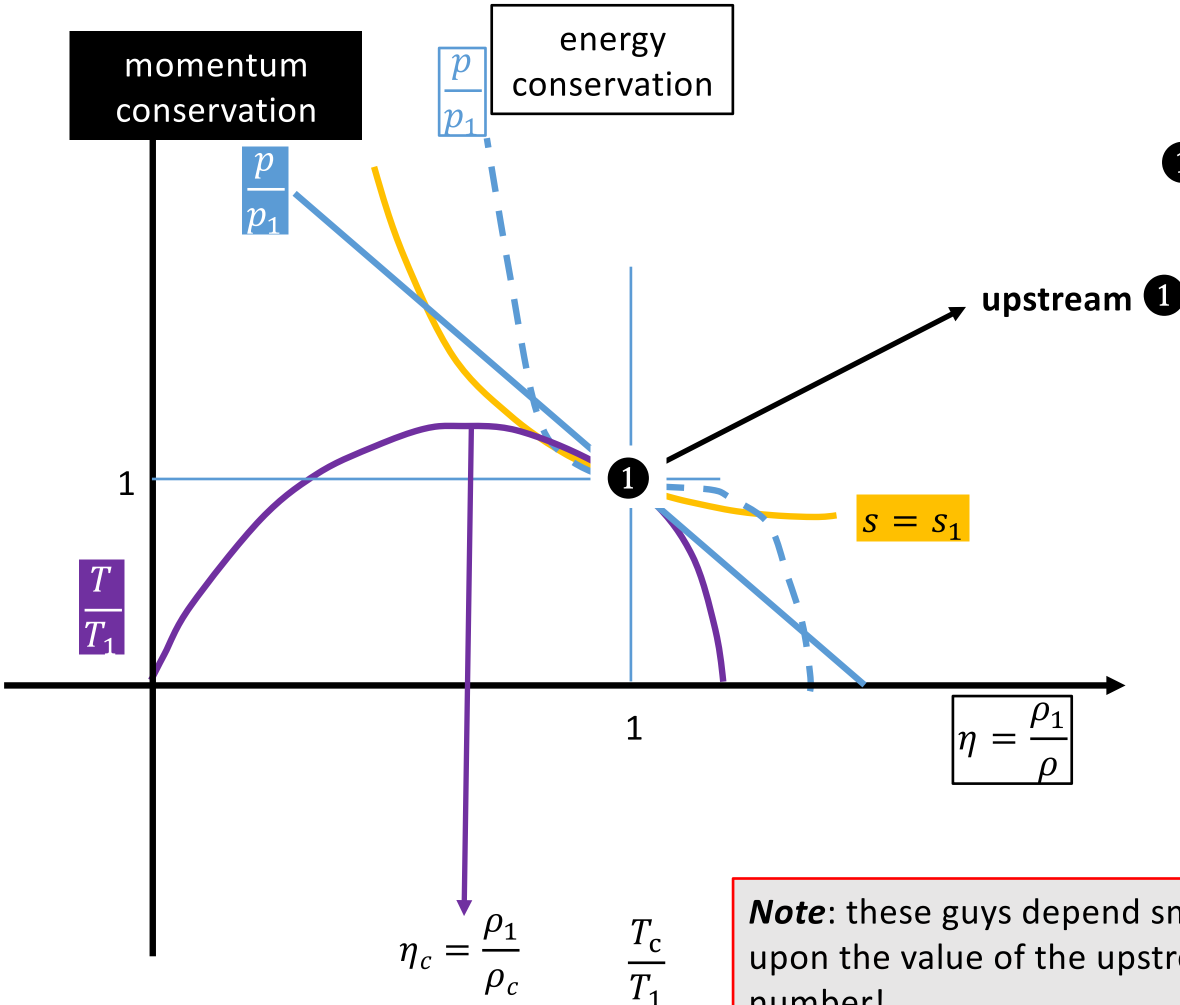

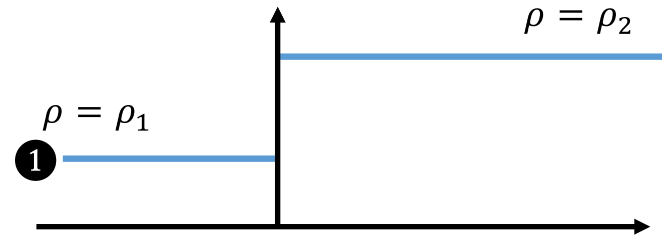

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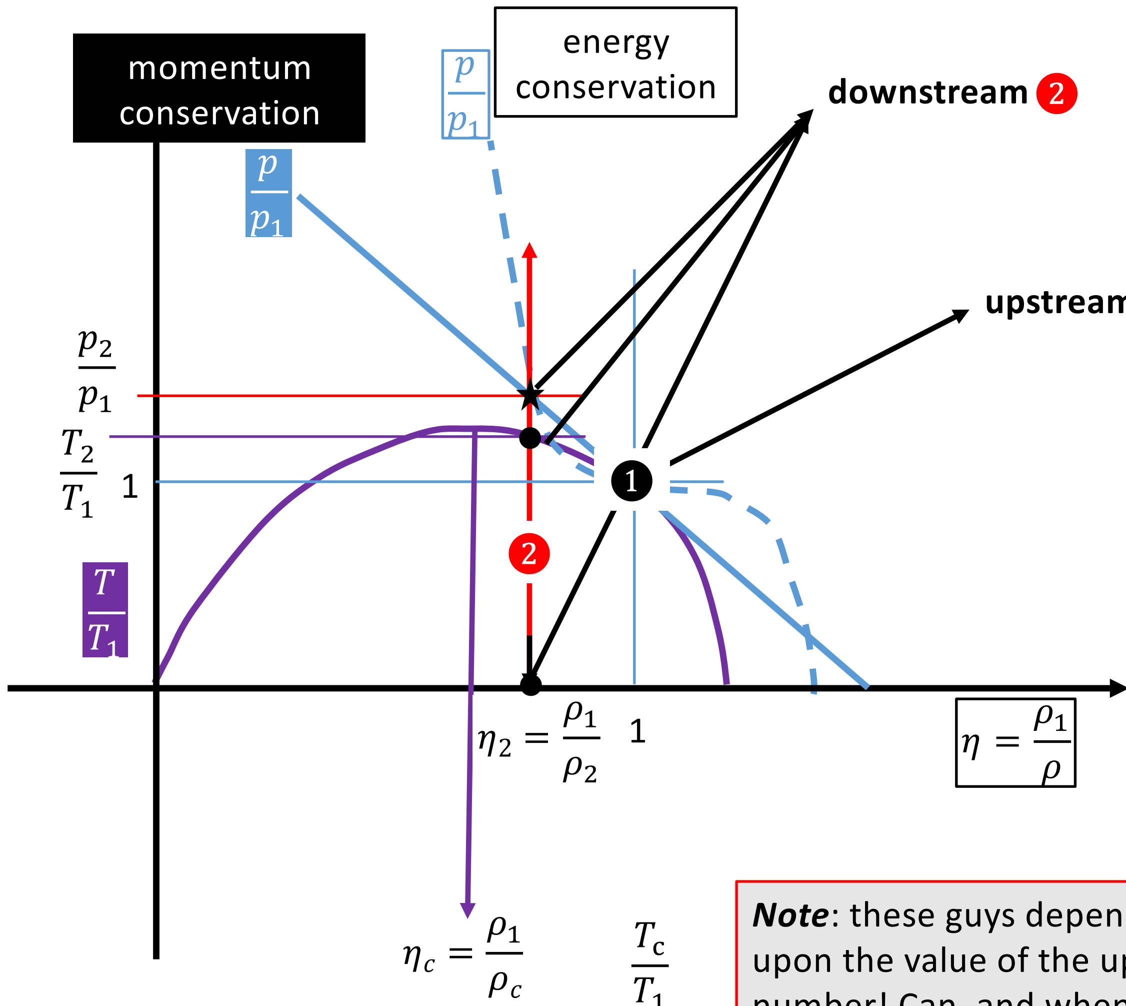
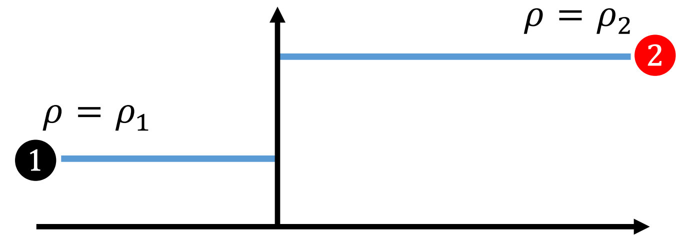


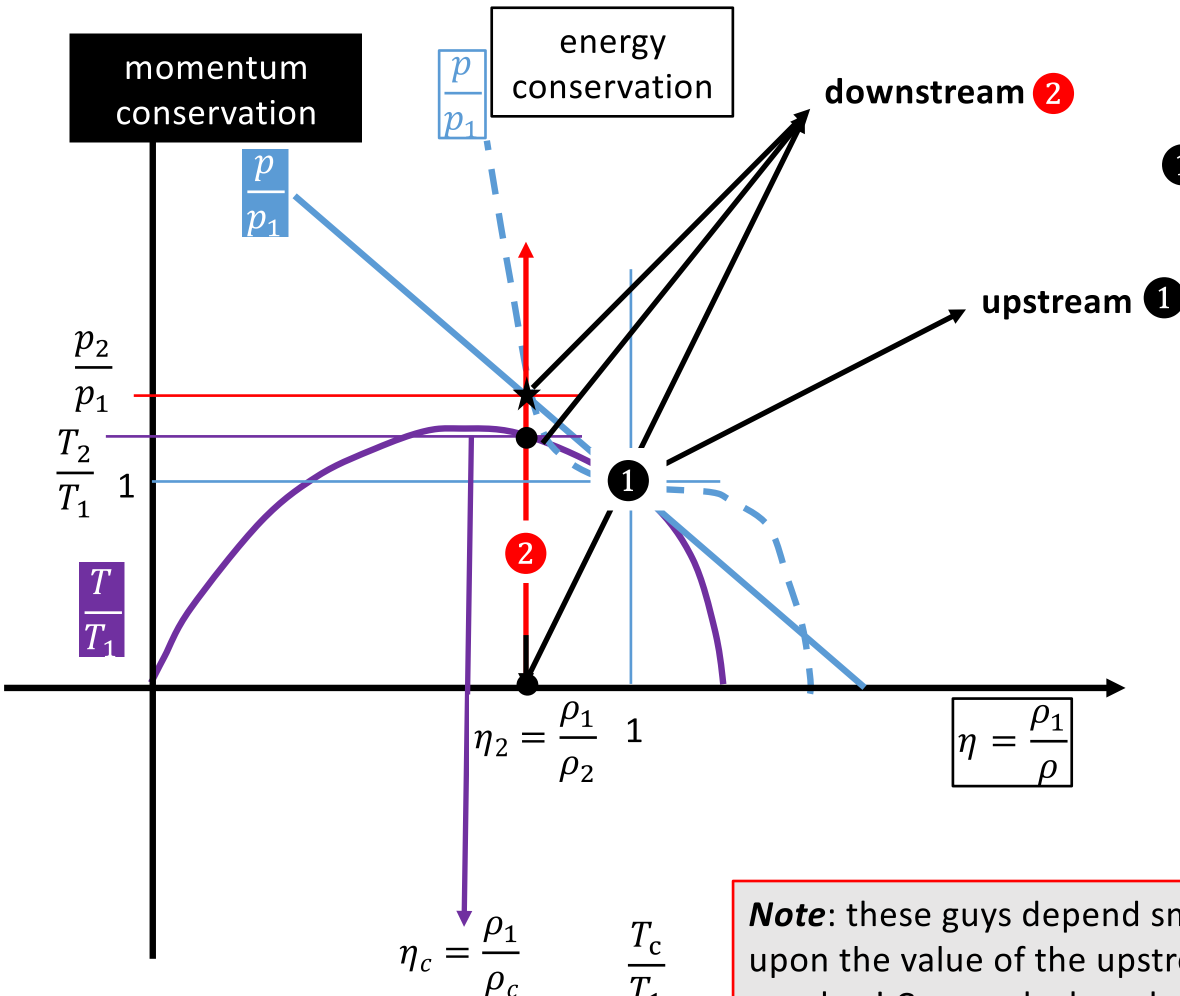


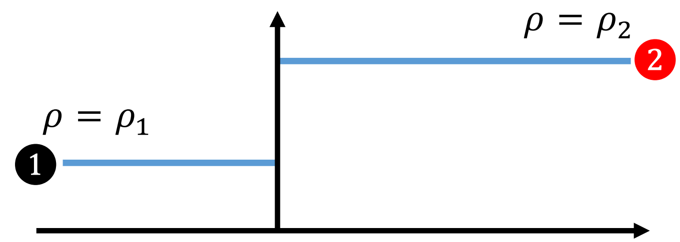
For a ***given***

$$M^2 = \frac{\rho_1 u_1^2}{\gamma p_1}$$

$$\frac{1}{\eta_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$$

Note: these guys depend smoothly upon the value of the upstream Mach number! Can, and when does, $\rho_2 = \rho_c$? What happens ***then***?!





Yes!

And ***nothing,*** unless the shock is radiating and/or conducting heat!

$$\eta_c = \frac{\gamma + 1}{3\gamma - 1} = 2/3$$

$$M_c^2 = \frac{3\gamma - 1}{\gamma(3 - \gamma)} = 1.8$$

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$$E = 3P$$

$$\mathcal{M} = \rho u$$

$$\mathcal{P} = \rho u^2 + p$$

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$$\mathcal{L} = \mathcal{M}\left(\frac{1}{2}u^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho}\right) + F$$

$$\boldsymbol{F} = \boldsymbol{F}'$$

$$\mathbb{P} = \mathbb{P}'$$

$$\nabla \cdot \boldsymbol{F} = \kappa \left[4\sigma_R T^4 - cE\right]$$

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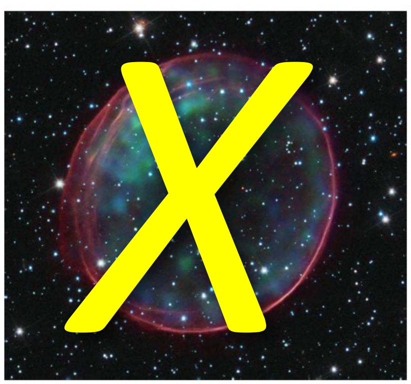

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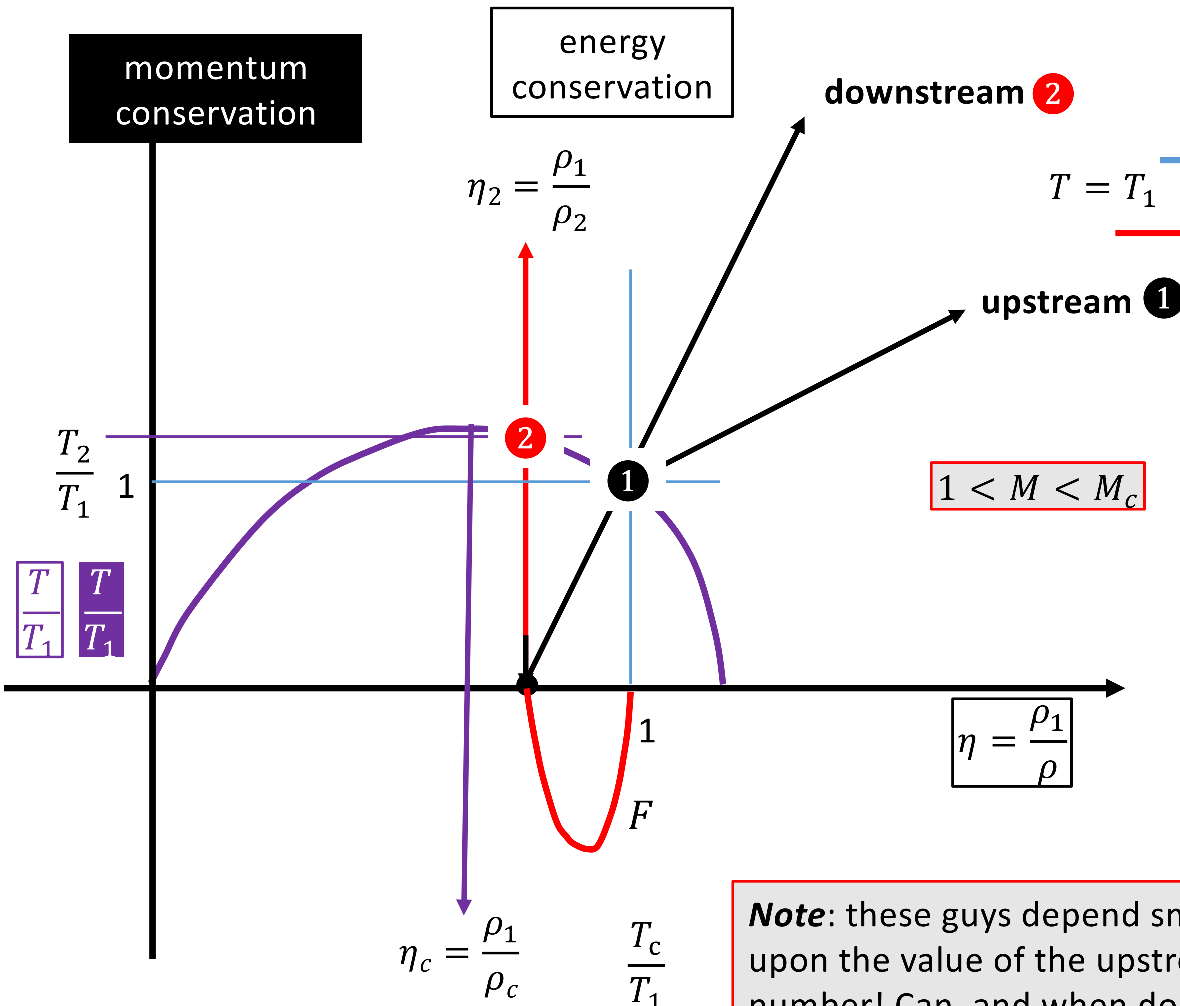
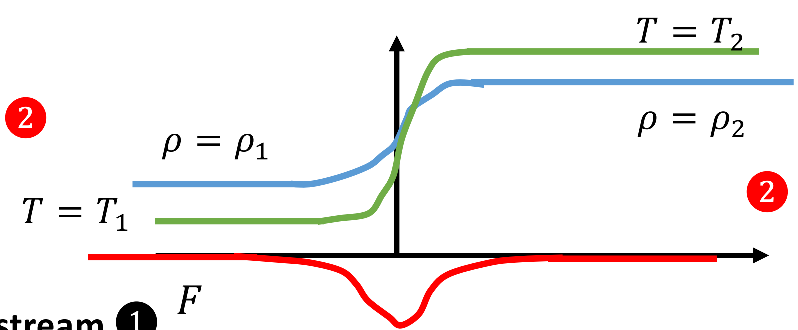
Optically-thick, non-LTE,
radiation pressure neglected, limit.





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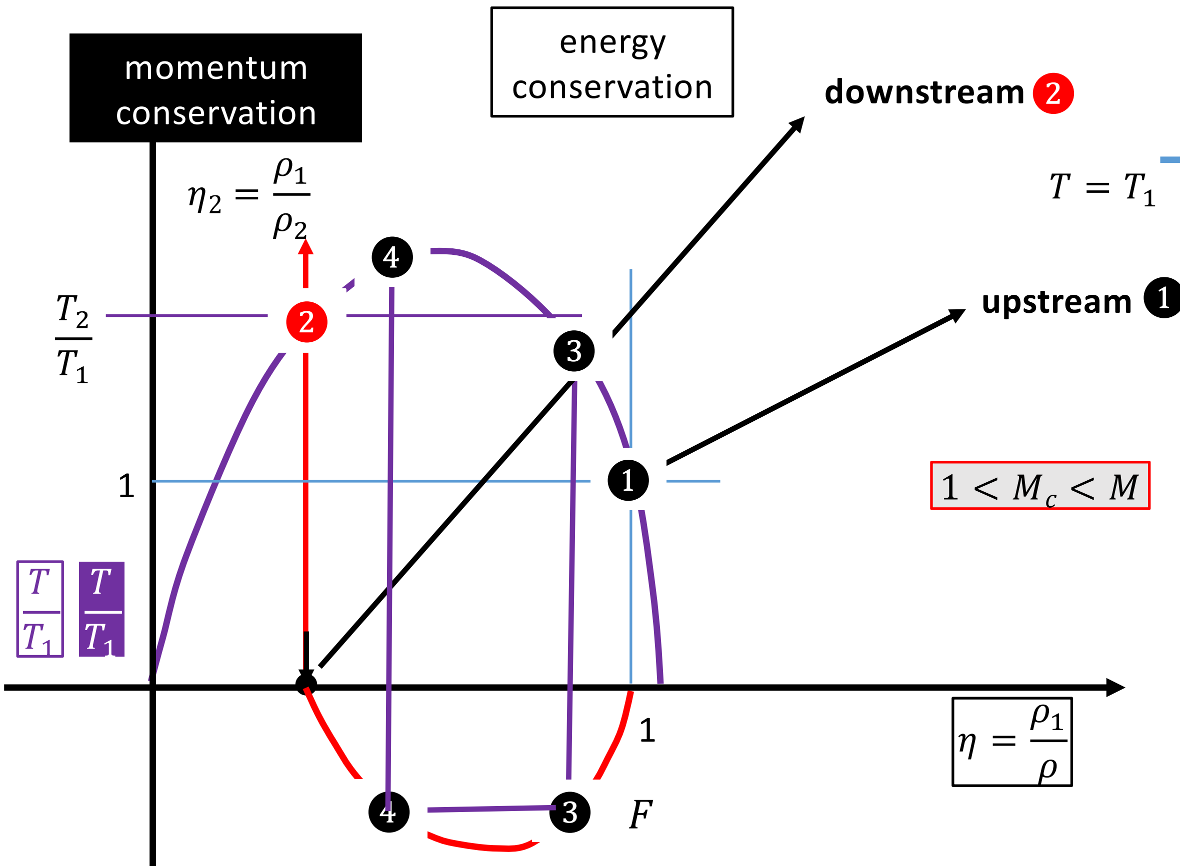

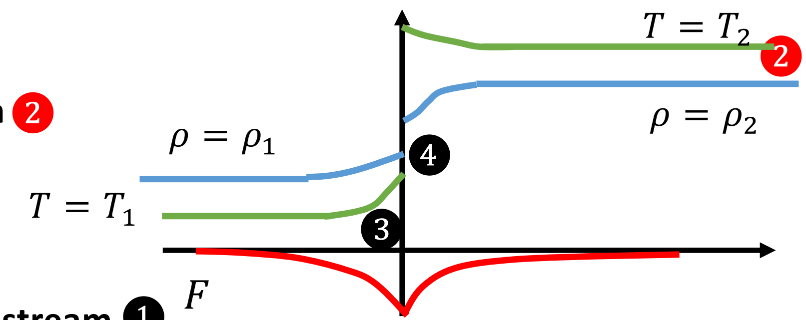

$\eta_c = \frac{\rho_1}{\rho_c}$ $\frac{T_c}{T_1}$

Yes!

And ***nothing,*** unless the shock is radiating and/or conducting heat!

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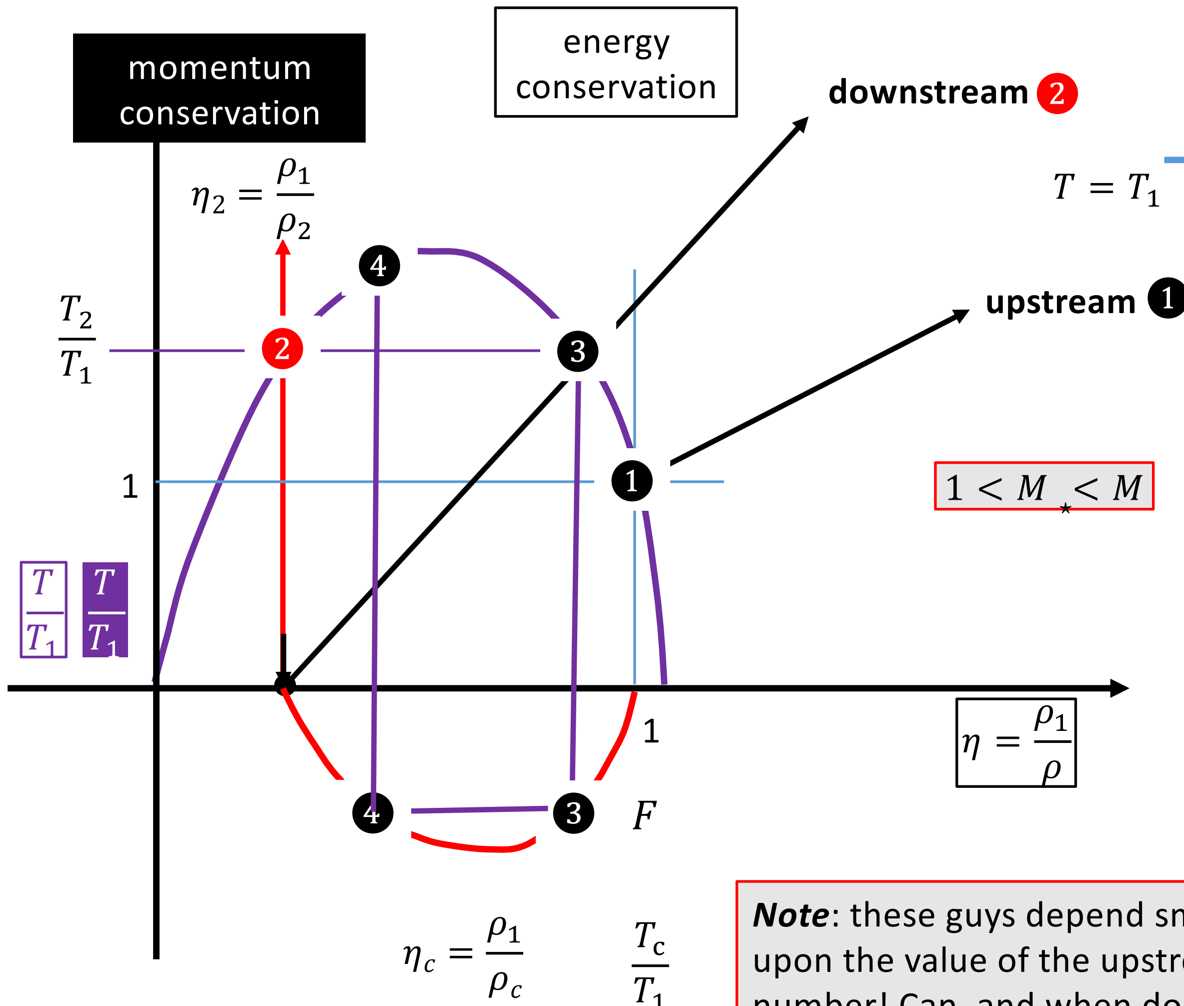

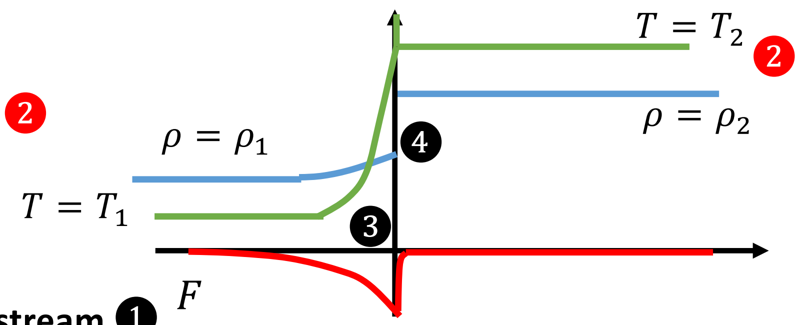

$1 < M_\star < M$

Yes!

And ***nothing,*** unless the shock is radiating and/or conducting heat!

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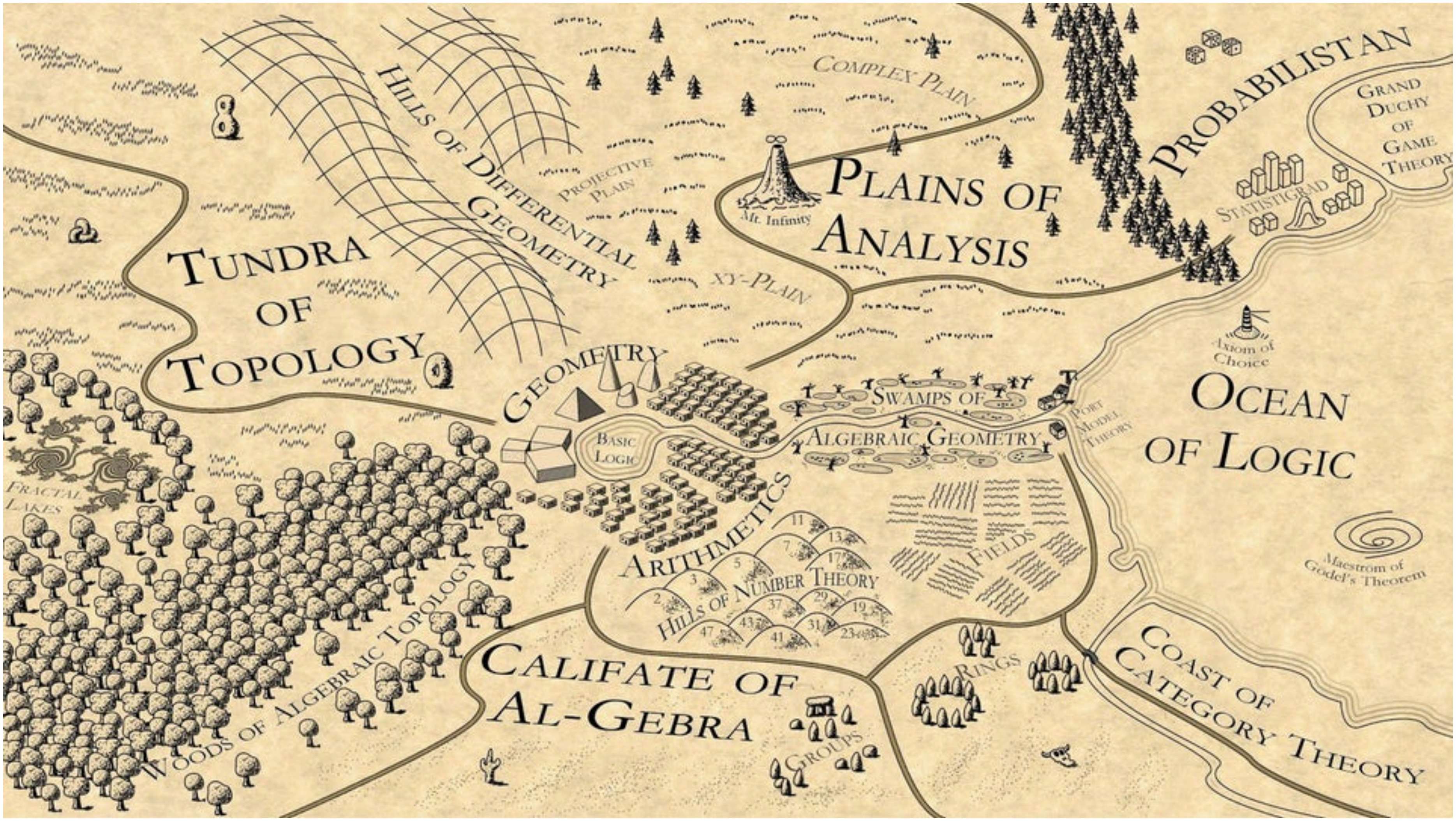

Complex Plain
Probabilistan
Grand Duchy of Game Theory
Hills of Differential Geometry
Projective Plain
Plains of Analysis
Mt. Infinity
Statistigrad
Tundra of Topology
XY-Plain
Axiom of Choice
Ocean of Logic
Geometry
Swamps of Algebraic Geometry
Port Model Theory
Basic Logic
Fractal Lakes
Arithmetics
Fields
Hills of Number Theory
Maelström of Gödel's Theorem
Woods of Algebraic Topology
Califate of Al-Gebra
Rings
Coast of Category Theory
Groups

We know F, T^4, κ as functions of the variable η.
Now what?

$$\tau = \int_0^z \mathrm{d}z' \, \kappa(z')$$

$$\frac{\mathrm{d}^2 F}{\mathrm{d}\tau^2} = 4\sigma_R \frac{\mathrm{d}T^4}{\mathrm{d}\tau} + 3F$$

$$F = 2\sigma_R \int_{-\infty}^{\infty} \mathrm{d}t \, e^{-\sqrt{3}|\tau - t|} \, \frac{\mathrm{d}T^4}{\mathrm{d}t}$$

We only know T^4 as functions of the variable η.
Now what?

$$\frac{\mathrm{d}F}{\mathrm{d}z} = \kappa[4\sigma_R T^4 - cE]$$

$$\frac{1}{3}\frac{\mathrm{d}E}{\mathrm{d}z} = -\frac{\kappa}{c}F$$

$$\frac{\mathrm{d}F}{\mathrm{d}\tau} = [4\sigma_R T^4 - cE]$$

$$\frac{1}{3}\frac{\mathrm{d}E}{\mathrm{d}\tau} = -\frac{1}{c}F$$

We know F, T^4, κ as functions of the variable η.
Now what?

$$\tau = \int_0^z \mathrm{d}z' \, \kappa(z')$$

$$\frac{\mathrm{d}^2 F}{\mathrm{d}\tau^2} = 4\sigma_R \frac{\mathrm{d}T^4}{\mathrm{d}\tau} + 3F$$

$$\frac{\mathrm{d}F}{\mathrm{d}\eta}\frac{\mathrm{d}^2\eta}{\mathrm{d}\tau^2} + \frac{\mathrm{d}^2 F}{\mathrm{d}\eta^2}\left(\frac{\mathrm{d}\eta}{\mathrm{d}\tau}\right)^2 = 3F + 4\sigma_R \frac{\mathrm{d}\eta}{\mathrm{d}\tau}\frac{\mathrm{d}T^4}{\mathrm{d}\eta}$$

$$F = 2\sigma_R \int_{-\infty}^{\infty} \mathrm{d}t \, e^{-\sqrt{3}|\tau - t|} \, \frac{\mathrm{d}T^4}{\mathrm{d}t}$$

We only know T^4 as functions of the variable η.
Now what?

Note: we need to watch out for the ***singularities*** where the coefficient in front of the highest derivative vanishes! They occur at the two end points far upstream and far downstream, and where $F'(\eta) = 0$. For the former, we can derive the leading order behavior of $\omega(\eta)$. The latter is to be avoided by inserting a discontinuity!

$$\tau = \int_0^z \mathrm{d}z' \, \kappa(z')$$

$$\frac{\mathrm{d}F}{\mathrm{d}\eta}\frac{\mathrm{d}^2\eta}{\mathrm{d}\tau^2} + \frac{\mathrm{d}^2F}{\mathrm{d}\eta^2}\left(\frac{\mathrm{d}\eta}{\mathrm{d}\tau}\right)^2 = 3F + 4\sigma_R \frac{\mathrm{d}\eta}{\mathrm{d}\tau}\frac{\mathrm{d}T^4}{\mathrm{d}\eta}$$

$$\frac{\mathrm{d}^2\eta}{\mathrm{d}\tau^2} \rightarrow \omega\frac{\mathrm{d}\omega}{\mathrm{d}\eta}, \qquad \left(\frac{\mathrm{d}\eta}{\mathrm{d}\tau}\right)^2 \rightarrow \omega^2, \qquad \frac{\mathrm{d}\eta}{\mathrm{d}\tau} \rightarrow \omega$$

$$\omega(\eta) = \frac{\mathrm{d}\eta}{\mathrm{d}\tau}$$

$$\mathrm{d}z = \frac{\mathrm{d}\eta}{\omega(\eta)\kappa(\eta)}$$

We solve one nonlinear first-order ODE on the interval $0 < \eta_2 \leq \eta \leq 1$ for $\omega(\eta)$. Problem solved!


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$$\mathcal{M} = \rho u$$

$$cE = cP = F$$

$$\nabla \cdot \boldsymbol{F} = \kappa \left[4\sigma_R T^4 \right]$$

$$\mathcal{P} = \rho u^2 + p$$

$$M^2 = \frac{\rho_1 u_1^2}{\gamma p_1} = \infty \qquad p_1 = T_1 = 0$$

$$\mathcal{L} = \mathcal{M}\left(\frac{1}{2}u^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho}\right) + F$$

$$\frac{p}{\rho} = \frac{\Re}{\mu} T$$

$$\boldsymbol{F} = \boldsymbol{F}'$$

$$\mathbb{P} = \mathbb{P}'$$

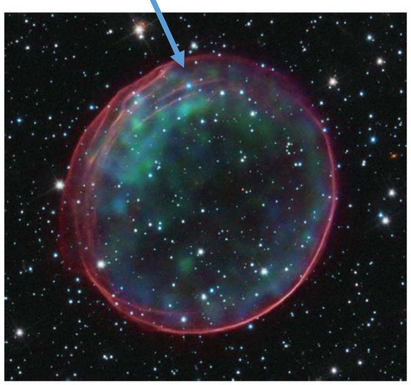
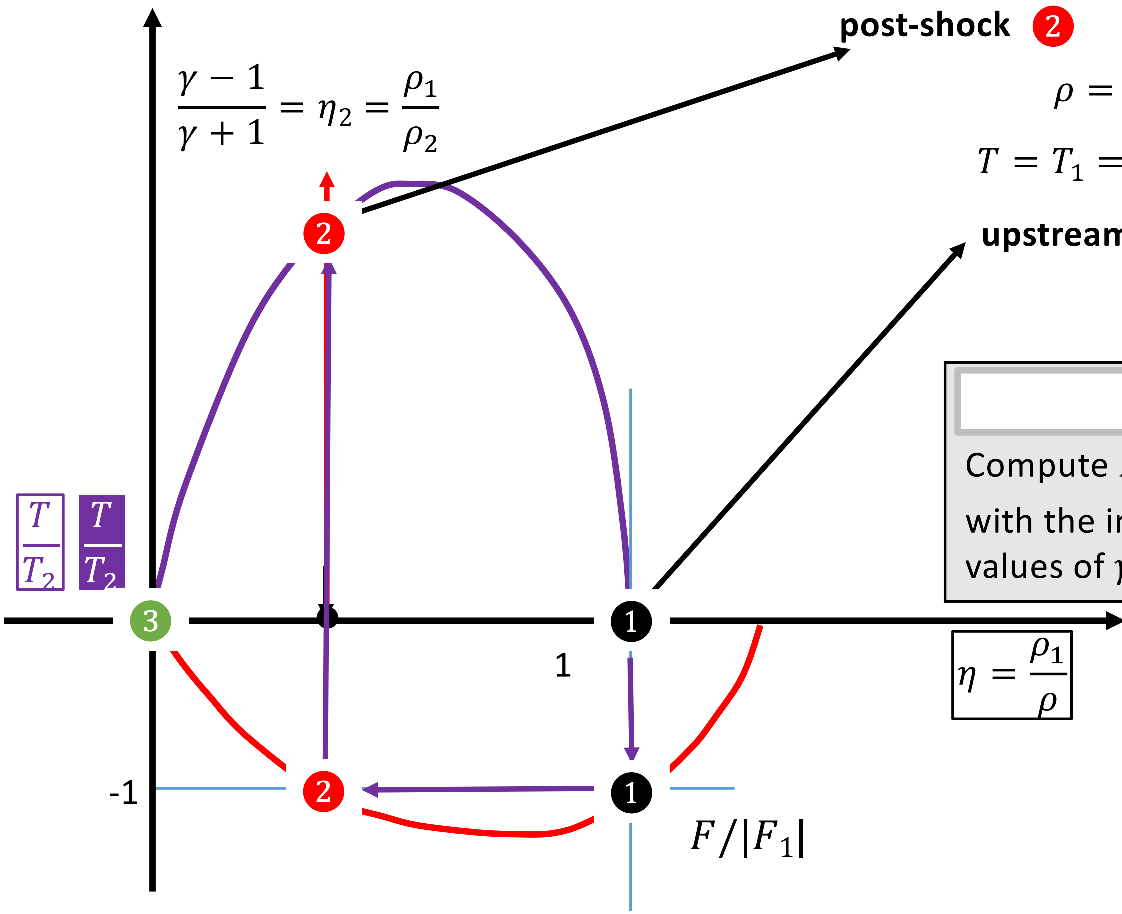
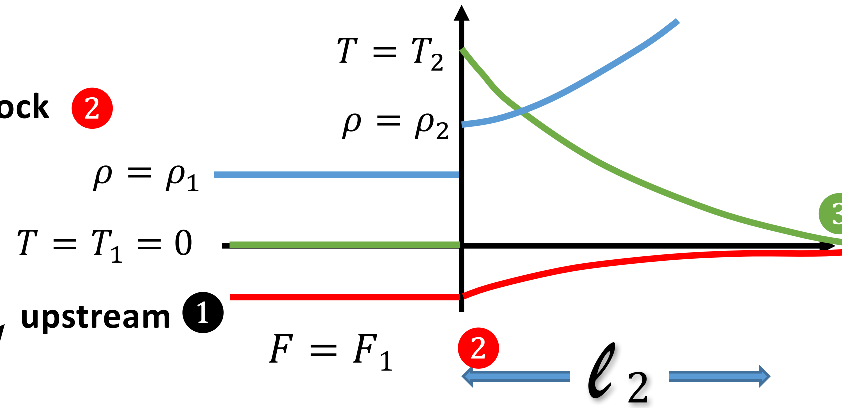


Figure 1.5: Onde de détonation produite par une supernova ayant pêté il y a plus de 400 ans (SNR 0509-67.5). Cette imagine combine des données optiques (rouge) et en rayon-X (vert-bleu). L'émission optique est associée au chauffage du milieu interstellaire par le passage de l'onde de choc. Image de la NASA, en domaine public.

Optically-thin, non-LTE,
radiation pressure neglected, limit.





Your HOMEWORK Assignment!

Compute F_1 and estimate ℓ_2. How do they compare with the image below? **Hint**: think carefully about the values of γ and κ.

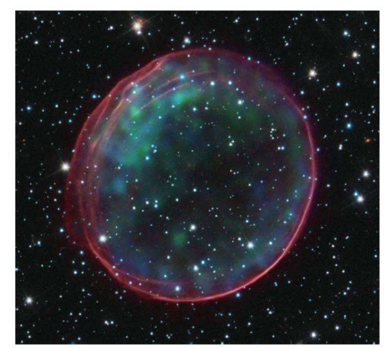


Figure 1.5: Onde de détonation produite par une supernova ayant pété il y a plus de 400 ans (SNR 0509-67.5). Cette imagine combine des données optiques (rouge) et en rayon-X (vert-bleu). L'émission optique est associée au chauffage du milieu interstellaire par le passage de l'onde de choc. Image de la NASA, en domaine public.

$$\mathcal{M} = \rho u$$

$$\mathcal{P} = \rho u^2 + p + P - \tau$$

$$\mathcal{L} = \mathcal{M}\left(\frac{1}{2}u^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho}\right) + \left(F - \frac{\alpha}{\nu + 1}\frac{\mathrm{d}}{\mathrm{dz}}T^{\nu+1}\right)$$

$$\boldsymbol{F} = \boldsymbol{F}' + \boldsymbol{u}[E' + \mathbb{P}'] + \cdots$$
$$\mathbb{P} = \mathbb{P}' + \frac{1}{c^2}[\boldsymbol{uF}' + \boldsymbol{F}'\boldsymbol{u}] + \cdots$$

$$\zeta = \frac{P_1}{p_1}$$

$$M^2 = \frac{\rho_1 u_1^2}{\gamma p_1}$$

Figure 1.5: Onde de détonation produite par une supernova ayant pêté il y a plus de 400 ans (SNR 0509-67.5). Cette imagine combine des données optiques (rouge) et en rayon-X (vert-bleu). L'émission optique est associée au chauffage du milieu interstellaire par le passage de l'onde de choc. Image de la NASA, en domaine public.

Give some other types of

SHOCKs

a try!

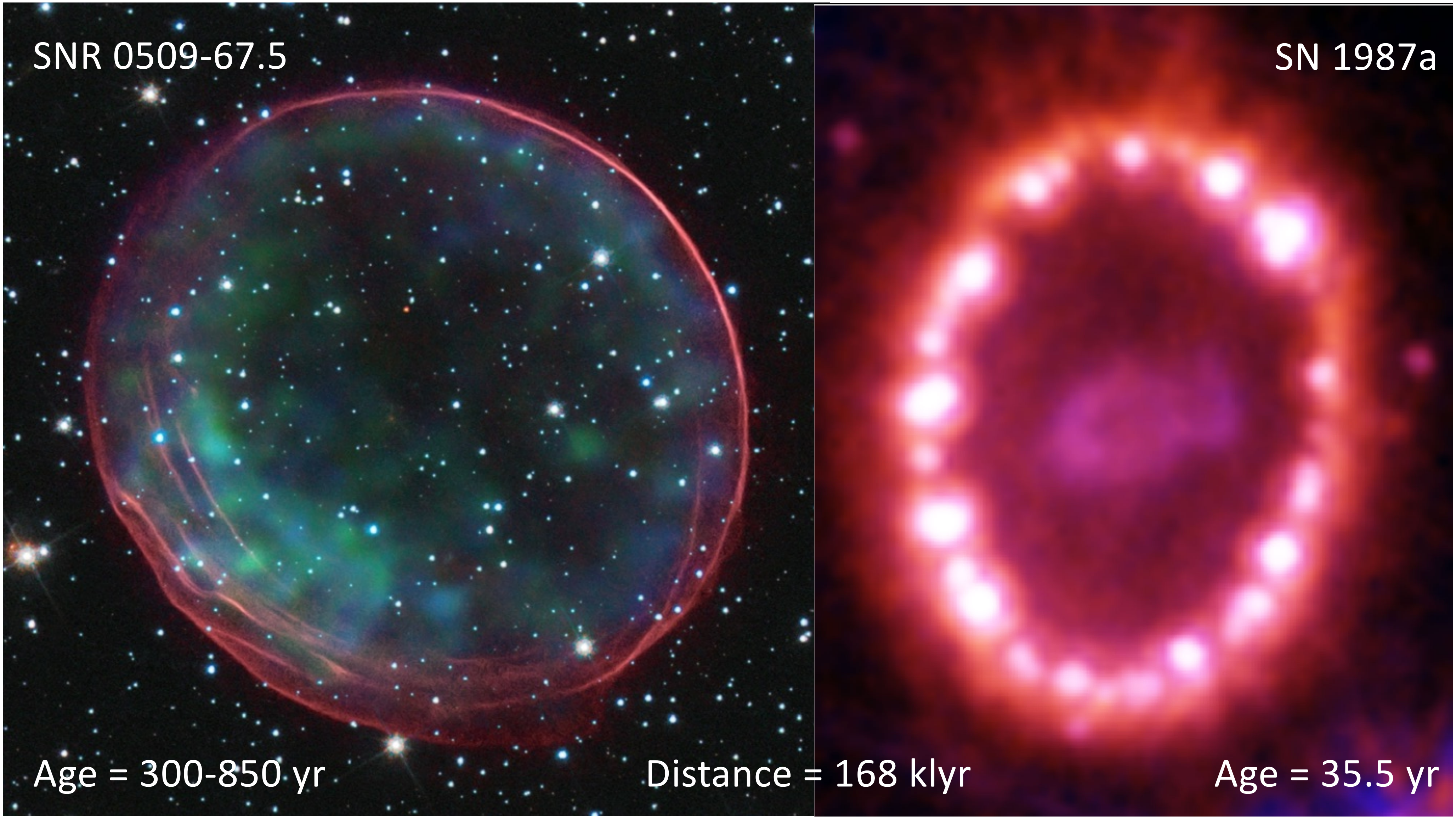
How could you include atomic ionization? Molecular dissociation? Magnetic Fields?

Astrophysical Radiation Magnetohydrodynamics

Being an Opera in Four Acts

ACT IV

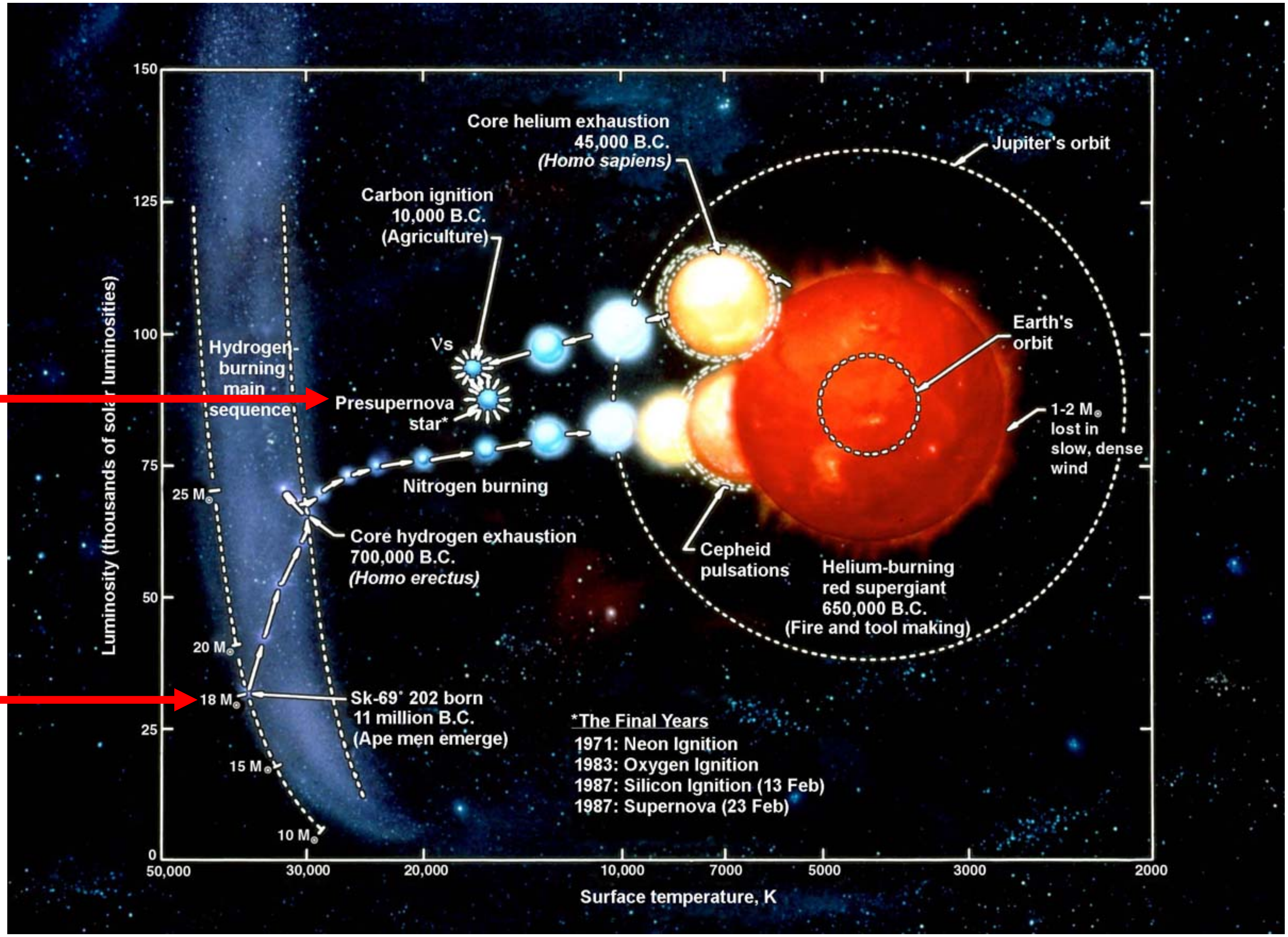
That I should love a bright particular star
And think to wed it, he is so far above me.
In his bright radiance and collateral light
Must I be comforted not in this sphere.

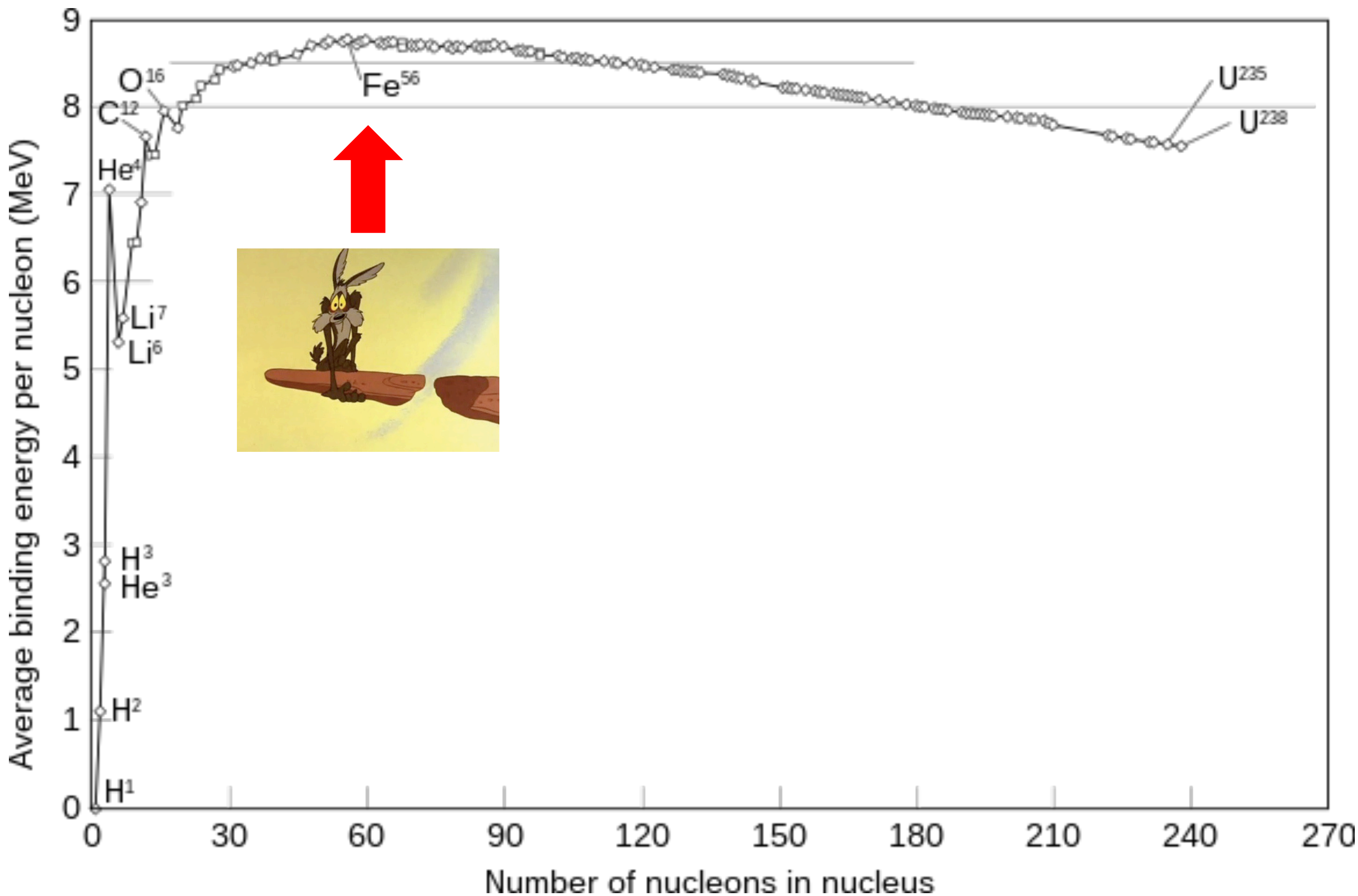

SNR 0509-67.5
SN 1987a
Age = 300-850 yr
Distance = 168 klyr
Age = 35.5 yr

SN 1987a

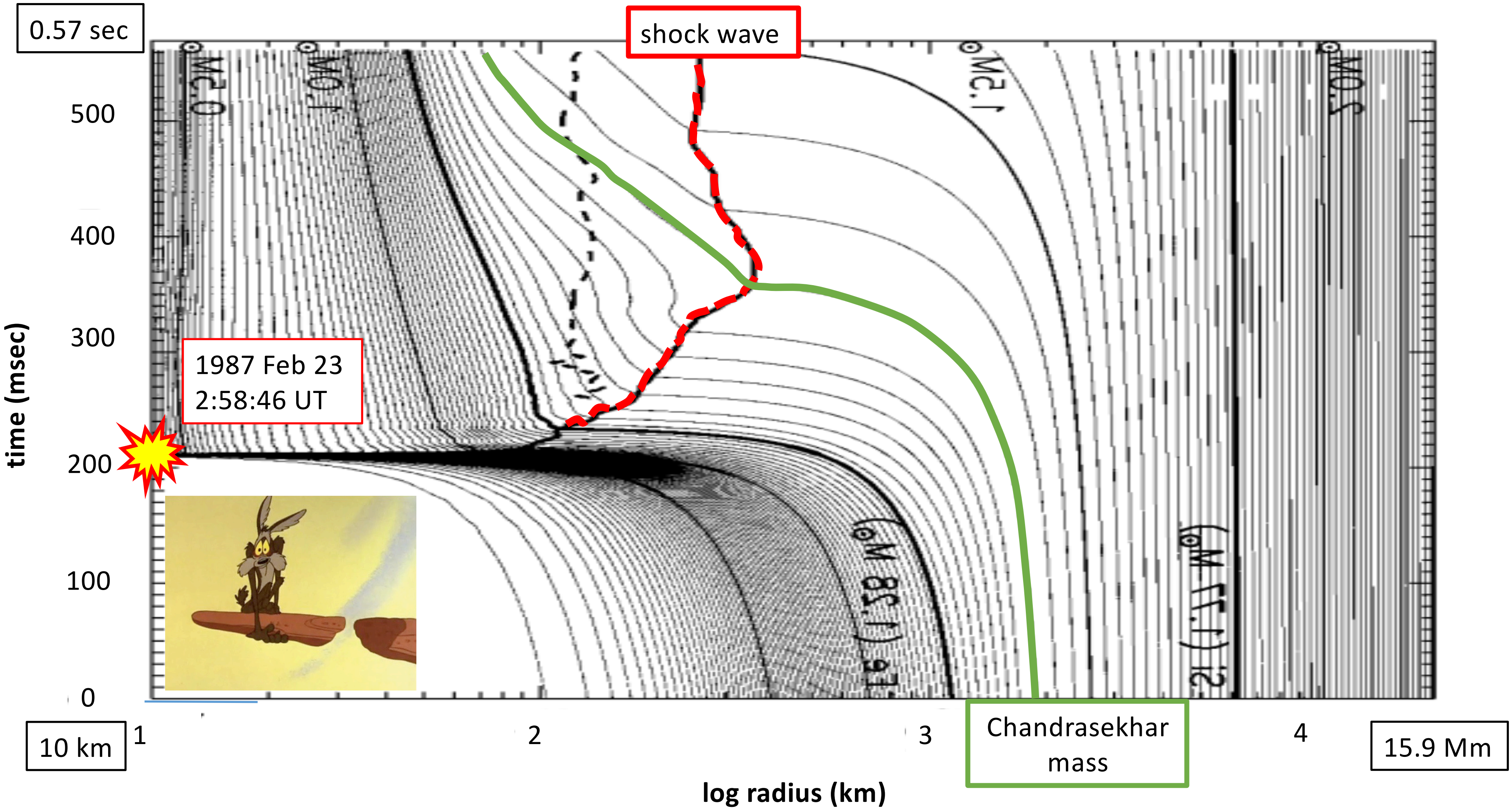
Stop here at
1987 Feb 23
2:58:46 UT

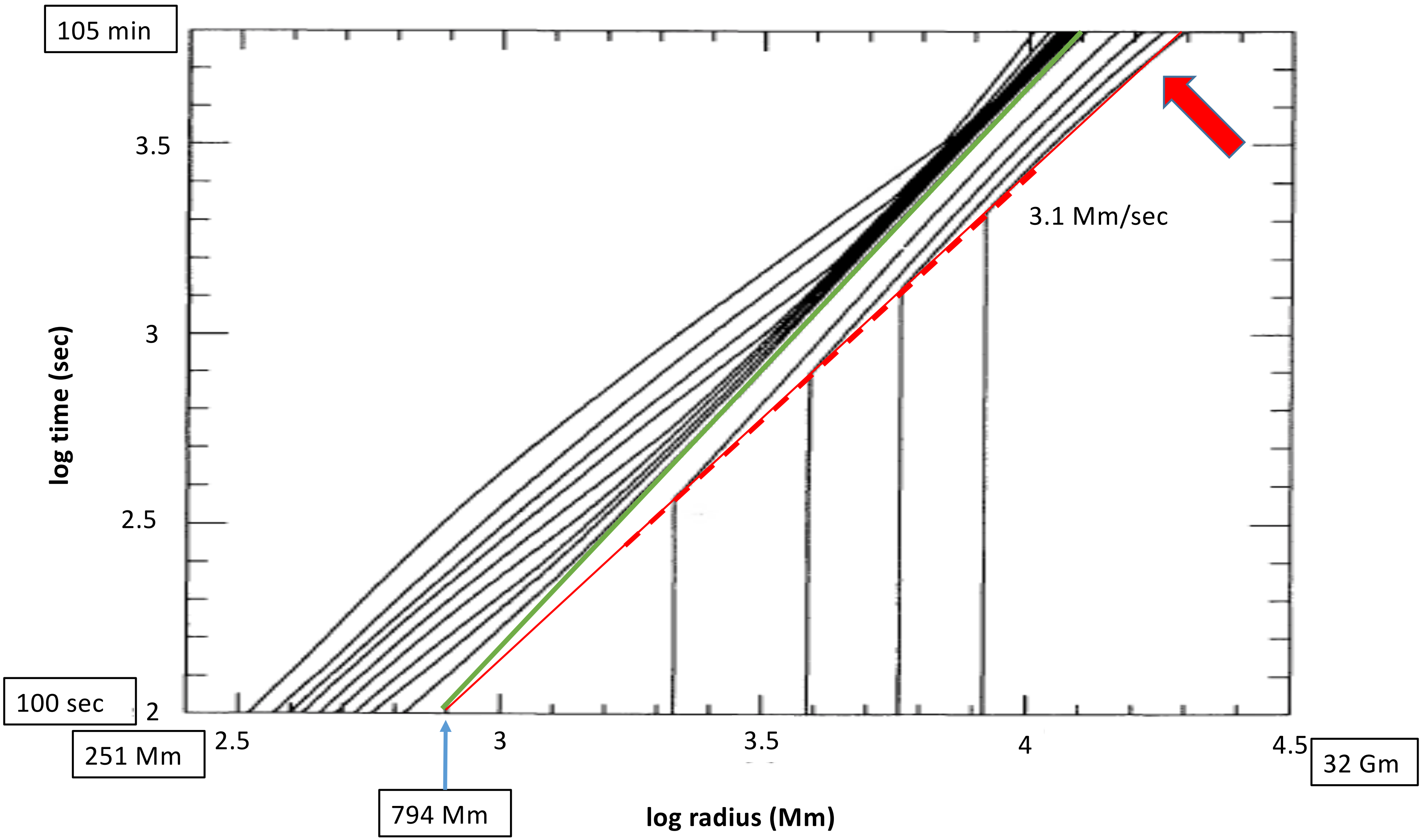
Start here at
-1.1×10^7 yr



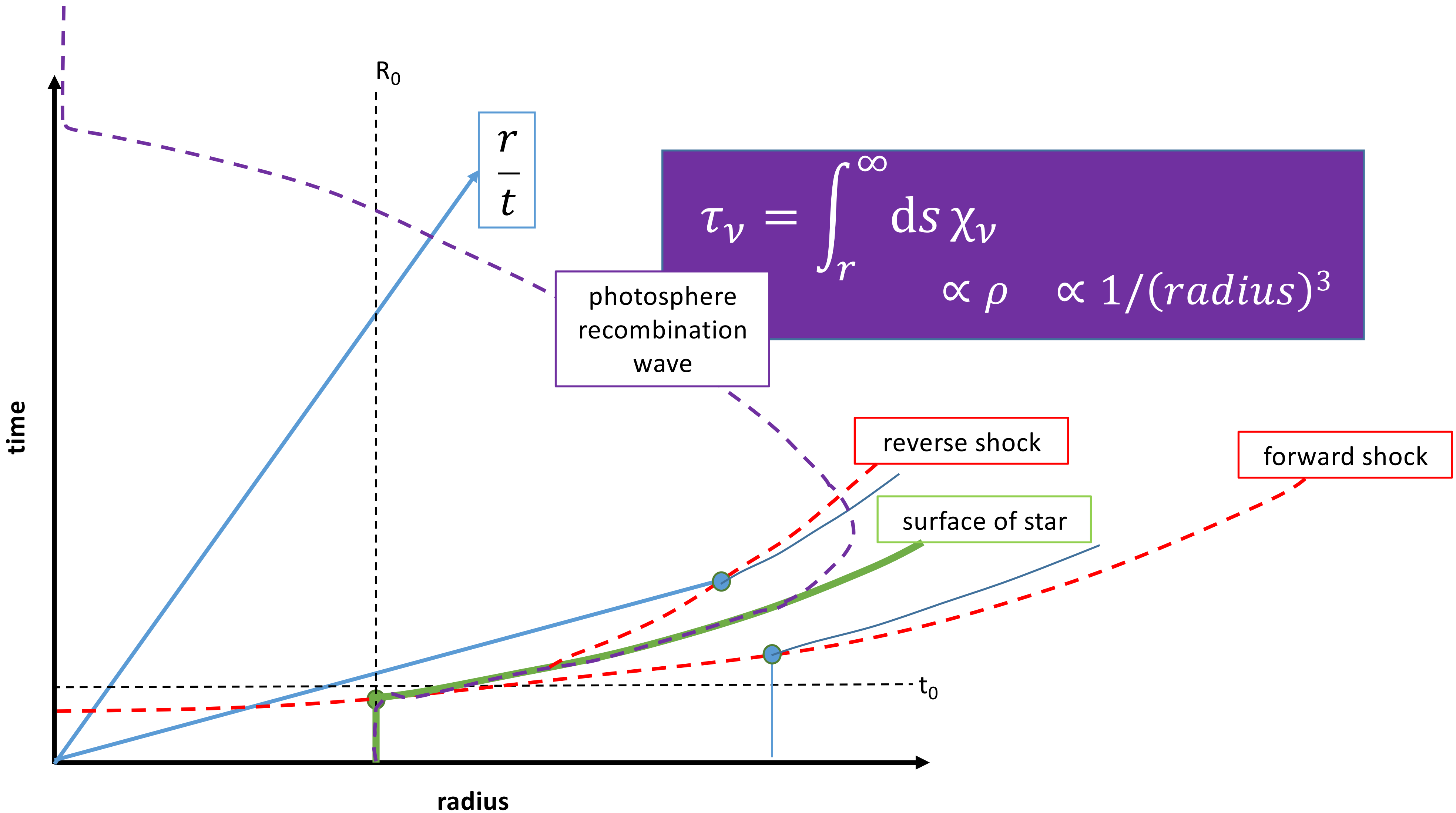


Average binding energy per nucleon (MeV)
Number of nucleons in nucleus
0
1
2
3
4
5
6
7
8
9
0
30
60
90
120
150
180
210
240
270
H1
H2
H3
He3
He4
Li6
Li7
C12
O16
Fe56
U235
U238

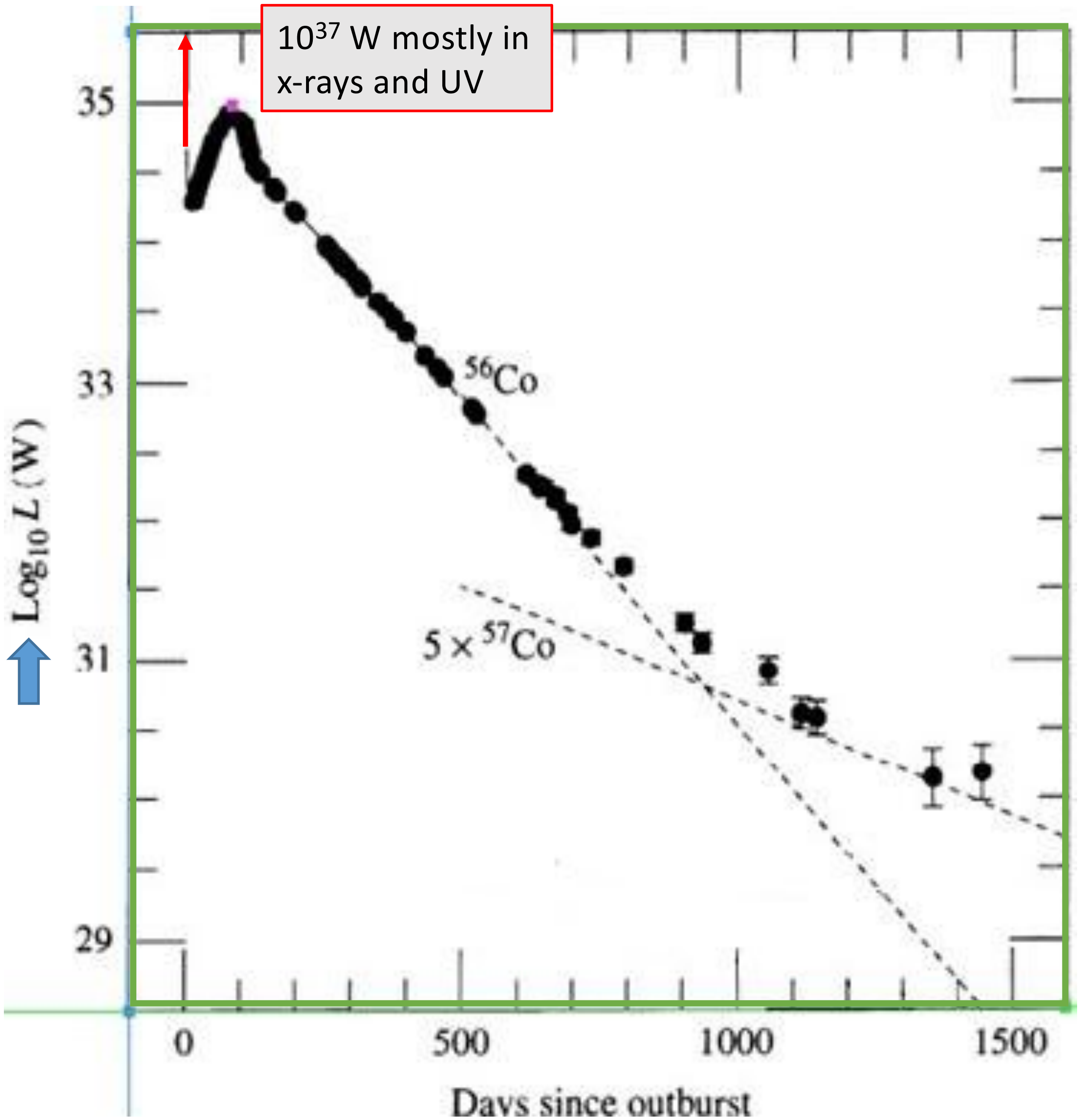

0.57 sec
shock wave
500
400
300
200
100
0
time (msec)
1987 Feb 23
2:58:46 UT
10 km
1
2
3
4
15.9 Mm
Chandrasekhar
mass
log radius (km)



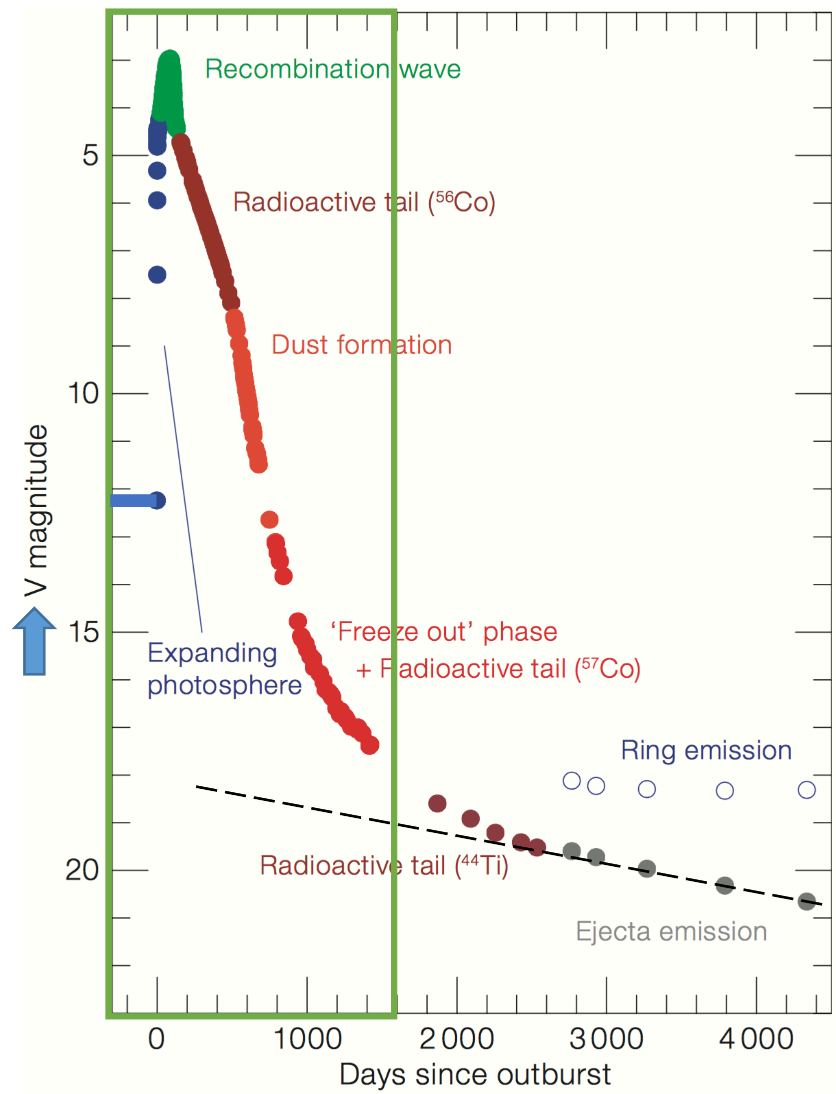
105 min
3.5
3
2.5
100 sec
2
251 Mm
2.5
3
3.5
4
4.5
32 Gm
794 Mm
log time (sec)
log radius (Mm)
3.1 Mm/sec



R_0
$\frac{r}{t}$
$\tau_\nu = \int_r^\infty \mathrm{d}s\, \chi_\nu$
$\propto \rho \quad \propto 1/(radius)^3$
photosphere recombination wave
reverse shock
forward shock
surface of star
t_0
time
radius



10^{37} W mostly in x-rays and UV
35
33
31
29
$Log_{10} L$ (W)
^{56}Co
$5 \times {}^{57}$Co
0
500
1000
1500
Days since outburst



Recombination wave
Radioactive tail (^{56}Co)
Dust formation
'Freeze out' phase
+ Radioactive tail (^{57}Co)
Expanding photosphere
Ring emission
Radioactive tail (^{44}Ti)
Ejecta emission
V magnitude
5
10
15
20
0
1000
2000
3000
4000
Days since outburst

LeQuatro

La primavera

$t < 10^3 yr$

$M_{ejecta} > M_{swept}$

$E_{ejecta} > E_{swept}$

Free Expansion

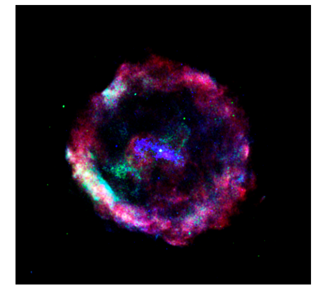
L'estate

$10^{3.7} yr < t < 10^{4.5}\ yr$

$M_{swept} > M_{ejecta}$

$E_{ejecta} > E_{swept}$

Adiabatic Expansion



L'autunno

$10^{4.7} yr < t < 10^{5.7}\ yr$

$M_{swept} > M_{ejecta}$

$E_{swept} > E_{ejecta}$

Snow Plough Expansion

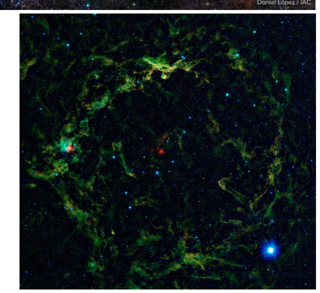
L'inverno

$t > 10^{5.9} yr$

$M_{swept} > M_{ejecta}$

$E_{swept} > KE_{ejecta}$

Background Merger



Did you notice how few actual numbers we needed to introduce? And then, only when we were determining transport coefficients!

$$\frac{\partial}{\partial t}\rho + \boldsymbol{\nabla}\cdot\rho\boldsymbol{u} = 0$$

$$\frac{\partial}{\partial t}\rho\boldsymbol{u} + \boldsymbol{\nabla}\cdot(p + \rho\boldsymbol{u}\boldsymbol{u}) = -\rho\,\boldsymbol{\nabla}\Phi + \boldsymbol{f}$$

$$\frac{\partial}{\partial t}\rho e + \boldsymbol{\nabla}\cdot(\rho e + p)\boldsymbol{u} = +\boldsymbol{u}\cdot\boldsymbol{\nabla}p + \rho T\dot{s}$$

These terms often come with built-in length and time scales, like GM and c, for example. They break the scaling symmetry of the magnetohydrodynamic equations.

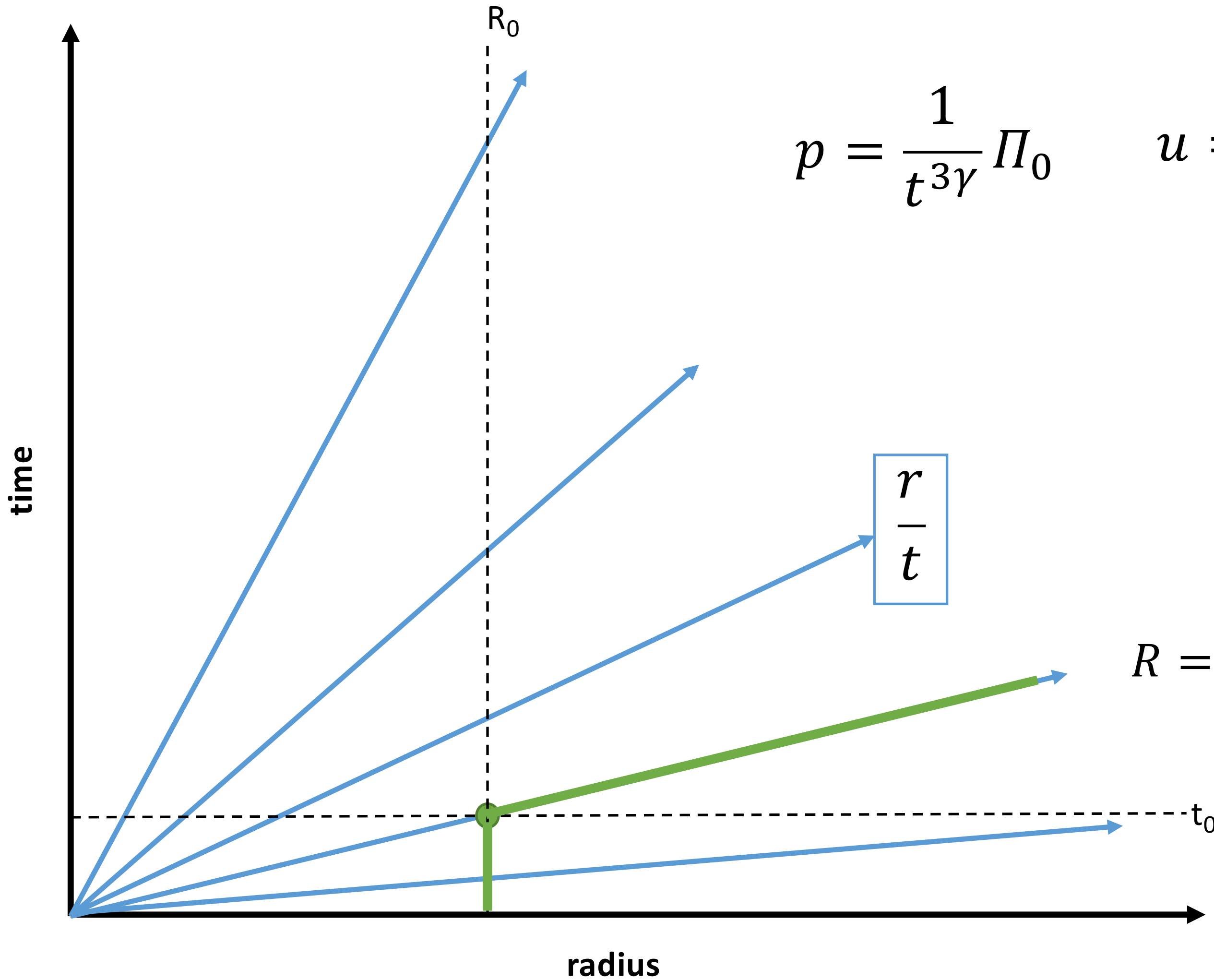
$$\frac{dr}{dt}$$

$$u \propto \frac{r}{t} \qquad \frac{p}{\rho} \propto \frac{r^2}{t^2} \qquad e \propto \frac{r^2}{t^2}$$

$$u = \frac{r}{t} \Rightarrow \rho = \frac{1}{t^3}\Omega(r/t) \quad p = \frac{1}{t^{3\gamma}}\Pi_0$$

Meh…

$$p = \frac{1}{t^{3\gamma}} \Pi_0 \qquad u = \frac{r}{t} = \frac{\mathrm{d}r}{\mathrm{d}t} \qquad \rho = \frac{1}{t^3}\Omega(r/t)$$



$$R = R_0 + \frac{R_0}{t_0}(t - t_0)$$

Suppose we try something just a little bit different?

$$\frac{\partial}{\partial t}\rho + \nabla \cdot \rho \boldsymbol{u} = 0$$

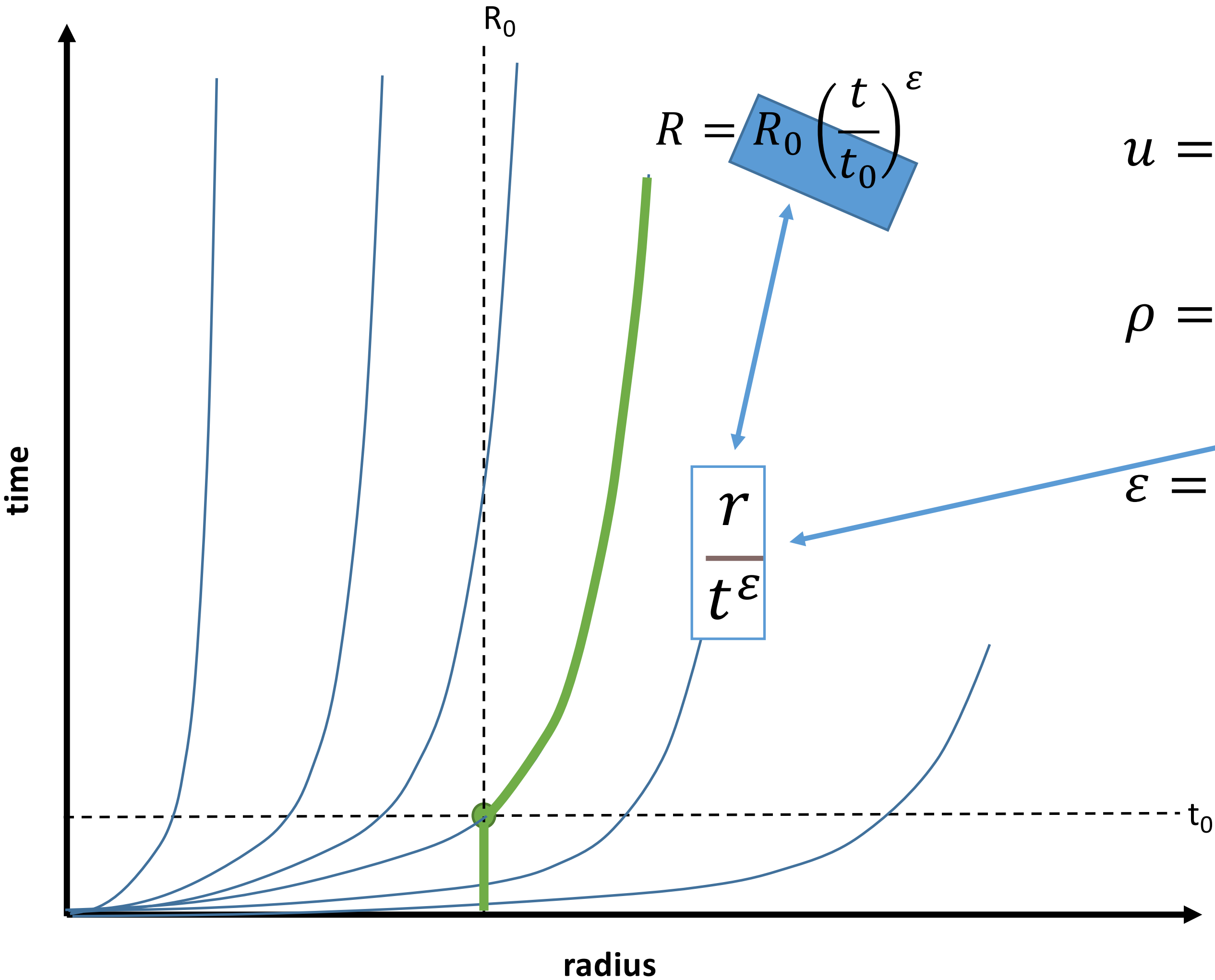
$$\frac{\partial}{\partial t}\rho \boldsymbol{u} + \nabla \cdot (p + \rho \boldsymbol{u}\boldsymbol{u}) = -\rho\, \nabla\Phi + \boldsymbol{f}$$

$$\frac{\partial}{\partial t}\rho e + \nabla \cdot (\rho e + p)\boldsymbol{u} = +\boldsymbol{u} \cdot \nabla p + \rho T \dot{s}$$

$$\frac{dr}{dt}$$

$$u = \varepsilon \frac{r}{t} \quad \Rightarrow \quad \frac{p}{\rho} \propto \frac{r^2}{t^2} \qquad \varepsilon = \frac{2}{3\gamma - 1}$$

Eureka!

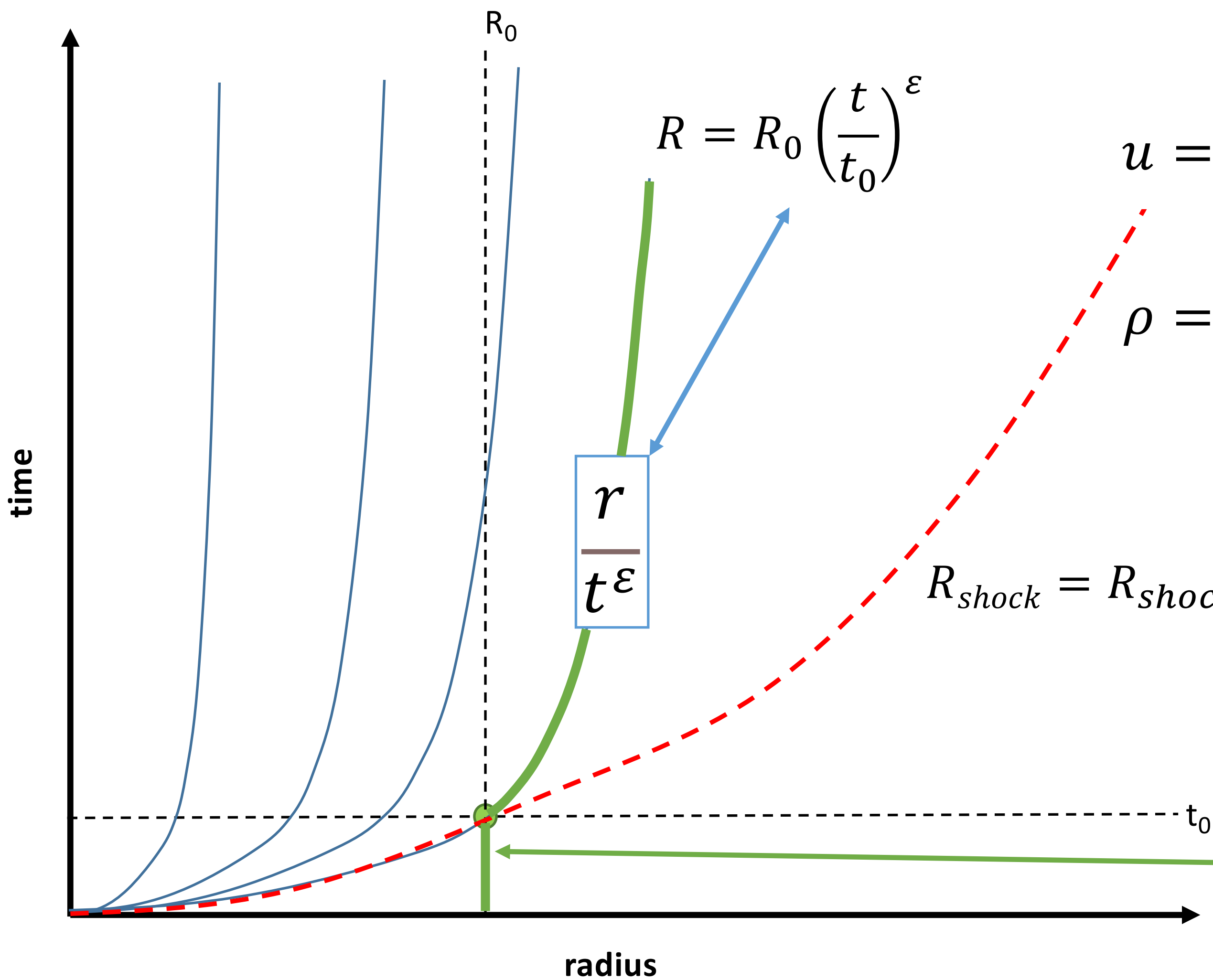


$$u = \varepsilon \frac{r}{t} = \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$\rho = \rho_0 \frac{\gamma + 1}{\gamma - 1} \frac{1}{t^{3\varepsilon}} \cdot \frac{r}{t^{\varepsilon}} \qquad \frac{p}{\rho} \propto \frac{r^2}{t^2}$$

$$\varepsilon = \frac{2}{3\gamma - 1}$$

Unbelievable!



$$R = R_0 \left(\frac{t}{t_0}\right)^{\varepsilon}$$

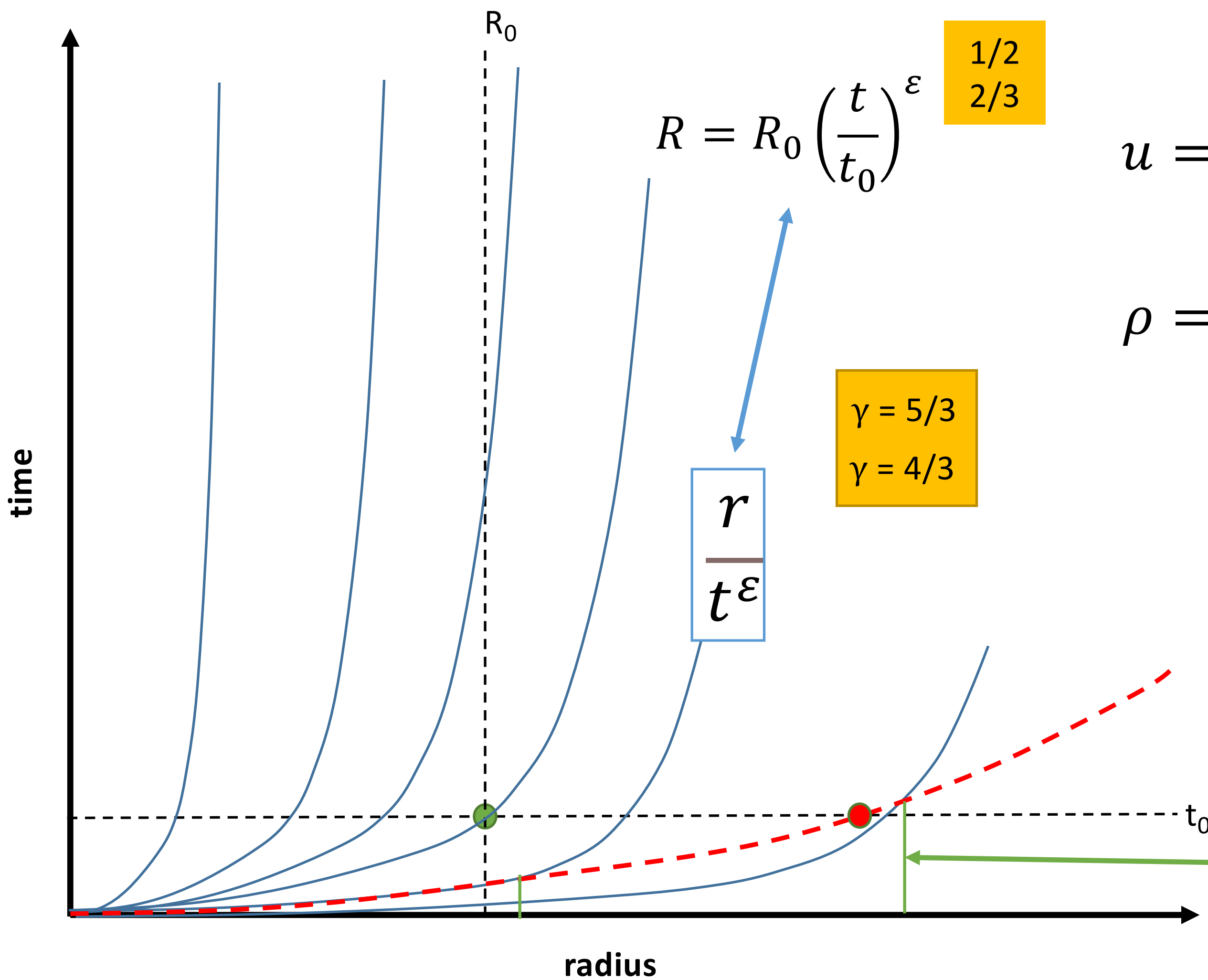
$$\frac{r}{t^{\varepsilon}}$$

$$u = \varepsilon \frac{r}{t} = \frac{\mathrm{d}r}{\mathrm{d}t} \qquad \varepsilon = \frac{2}{3\gamma - 1}$$

$$\rho = \rho_0 \frac{\gamma + 1}{\gamma - 1} \frac{1}{t^{3\varepsilon}} \cdot \boxed{\frac{r}{t^{\varepsilon}}}$$

$$R_{shock} = R_{shock_0} \left(\frac{t}{t_0}\right)^{(\gamma+1)\varepsilon/2}$$

$$\rho_\infty \propto \left(\frac{1}{r}\right)^{(7-\gamma)/(\gamma+1)}$$



You can't make this stuff up…

$$u = \varepsilon \frac{r}{t} = \frac{\mathrm{d}r}{\mathrm{d}t} \qquad \varepsilon = \frac{2}{3\gamma - 1}$$

$$\rho = \rho_0 \frac{\gamma + 1}{\gamma - 1} \frac{1}{t^{3\varepsilon}} \cdot \frac{r}{t^{\varepsilon}}$$

4
7

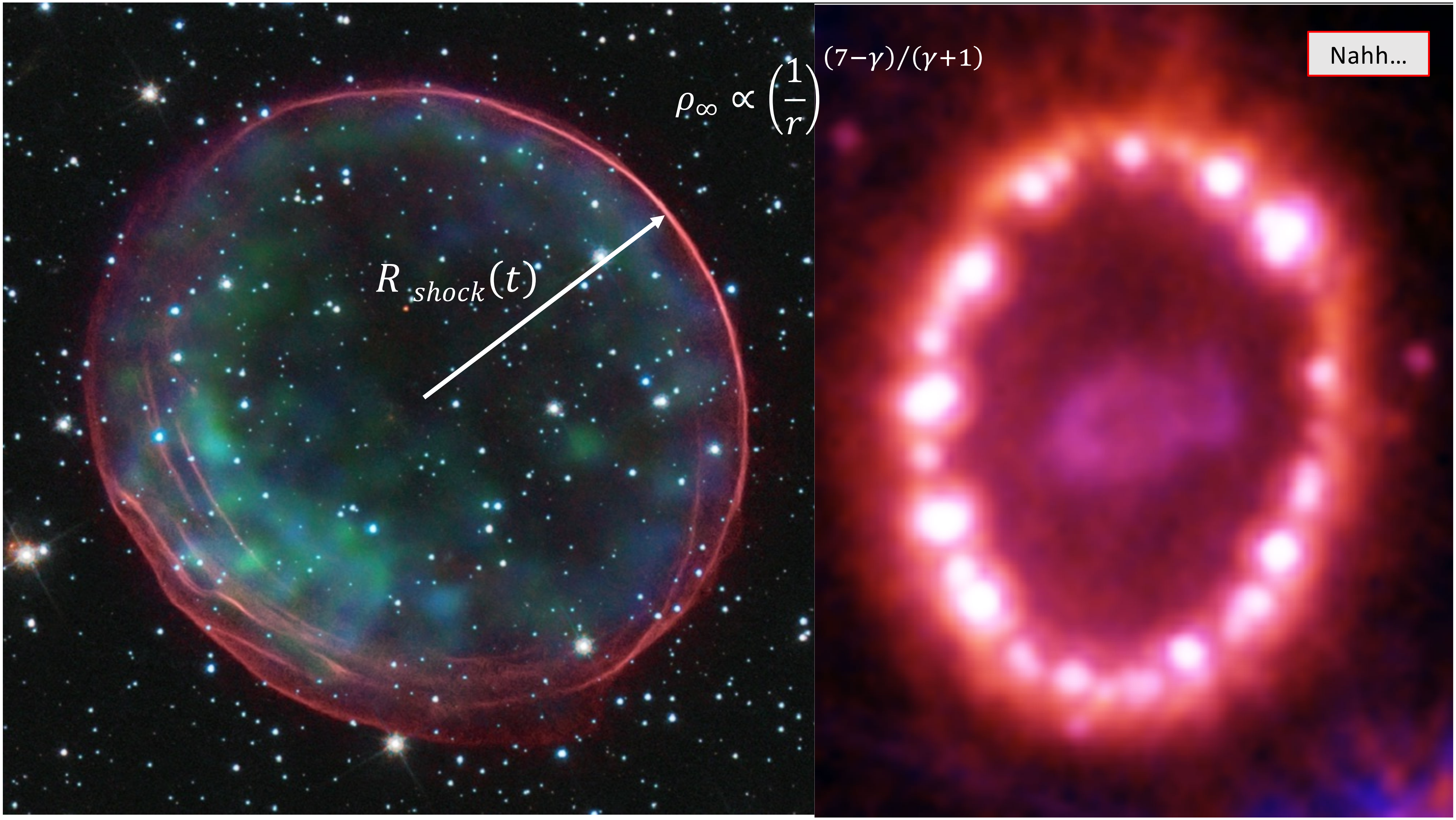
$$\eta_c = \frac{\gamma + 1}{3\gamma - 1}$$

$$R_{shock} = R_{shock_0} \left(\frac{t}{t_0}\right)^{(\gamma+1)\varepsilon/2}$$

2/3
7/9

$$\rho_\infty \propto \left(\frac{1}{r}\right)^{(7-\gamma)/(\gamma+1)}$$

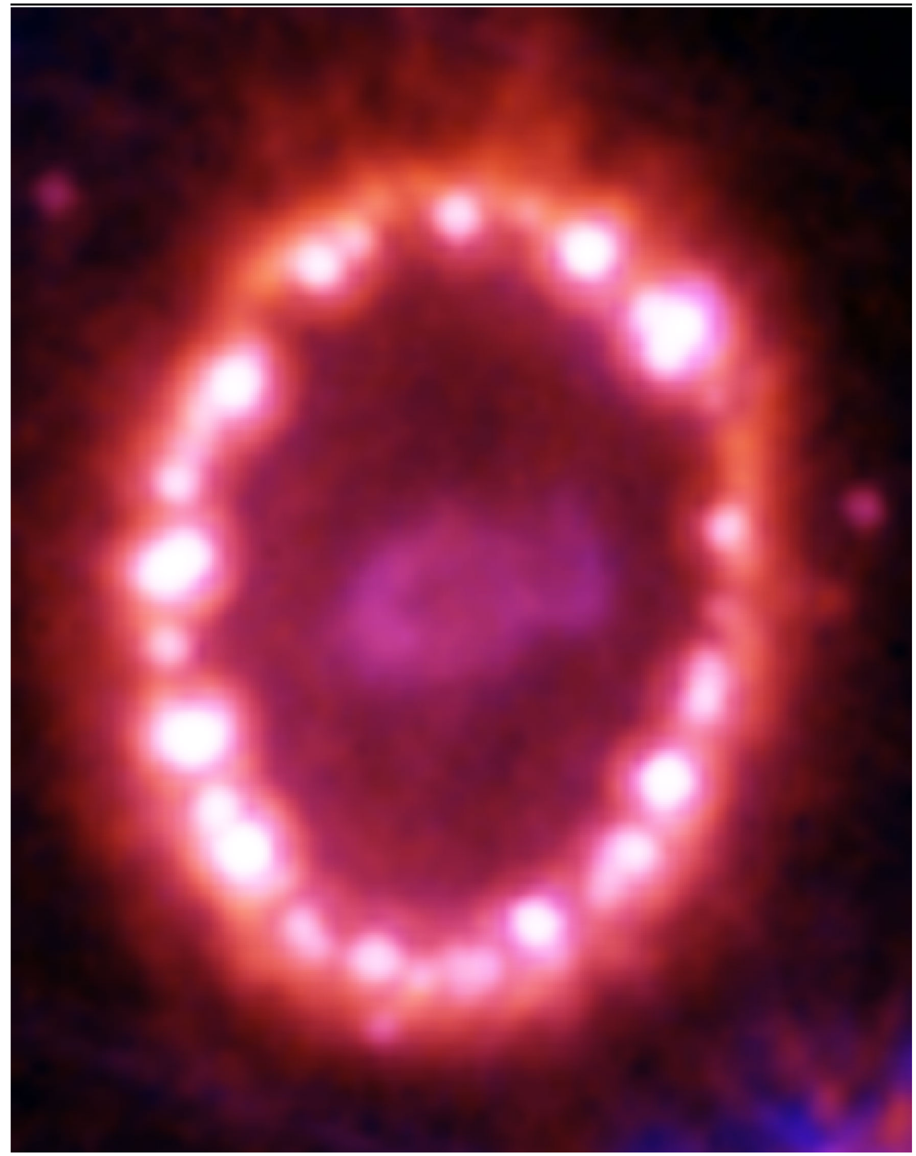
2
17/7


$R_{shock}(t)$
$\rho_\infty \propto \left(\frac{1}{r}\right)^{(7-\gamma)/(\gamma+1)}$
Nahh…

- It is ***cosmically unlikely*** that the density outside of a supernova cares about the ratio of specific heats---which has to be 4/3 at least early on---giving $r^{-17/7}$ (!!!)

- ***If*** the supernova were truly expanding adiabatically then we wouldn't see it at all---some photons are getting out

- ***Ergo***, we need a more realistic *energy equation* which incorporates radiative transfer ***and*** a more complicated *velocity profile*



$$\begin{cases} \frac{\partial}{\partial t}\rho + \boldsymbol{\nabla}\cdot\rho\boldsymbol{u} = 0 \\ \frac{\partial}{\partial t}\rho\boldsymbol{u} + \boldsymbol{\nabla}\cdot(p+\rho\boldsymbol{u}\boldsymbol{u}) = -\rho\,\boldsymbol{\nabla}\Phi + \boldsymbol{f} \\ \frac{\partial}{\partial t}\rho e + \boldsymbol{\nabla}\cdot(\rho e + p)\boldsymbol{u} = +\boldsymbol{u}\cdot\boldsymbol{\nabla}p + \rho T\dot{s} \end{cases}$$

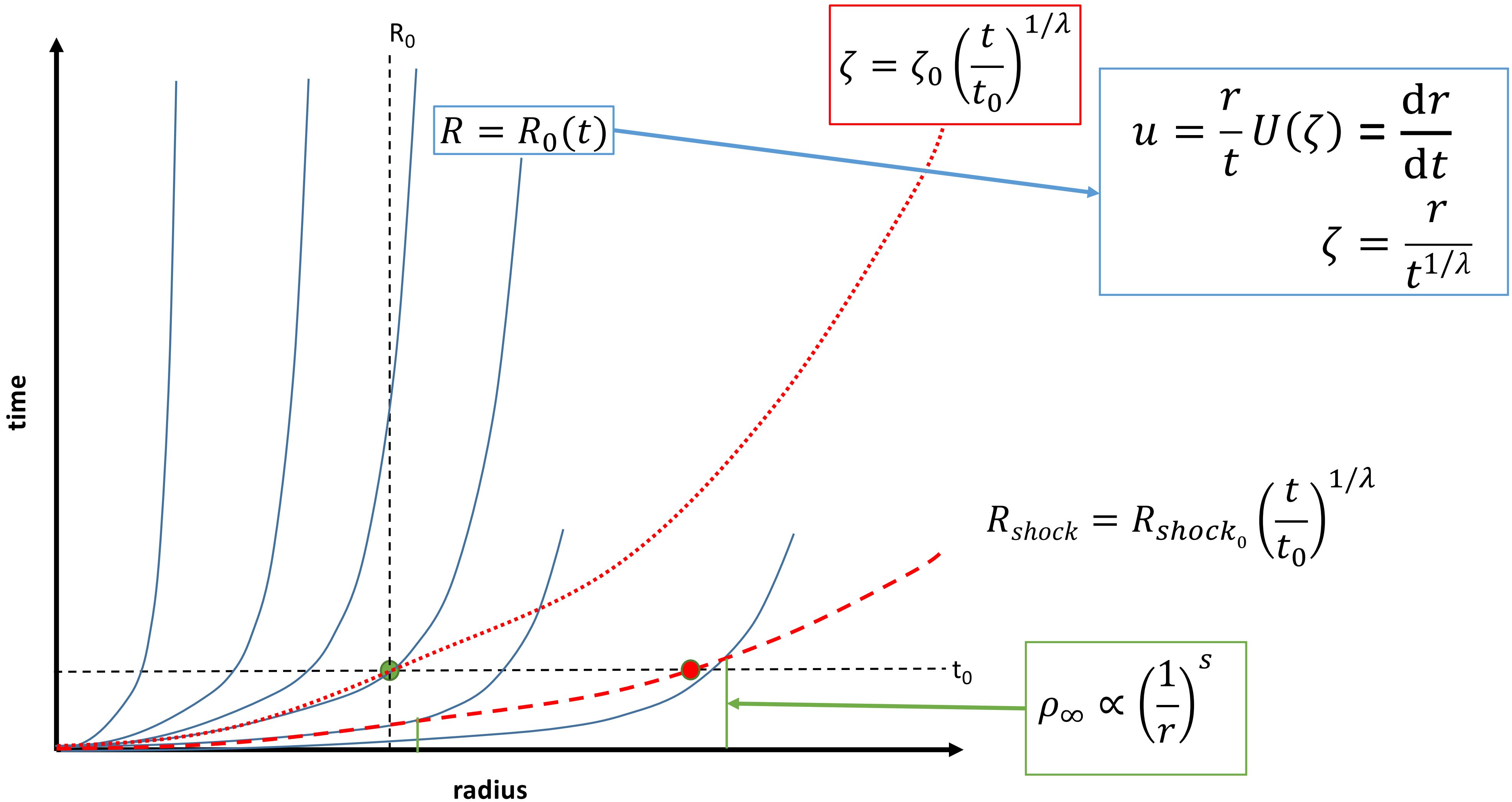
One can ***generalize*** the concept and push the idea just a little further. We call these ***intermediate asymptotics*** or ***similarity solutions***. Can we also accommodate ***gravity***, ***radiation*** and ***electromagnetism*** in this exercise?

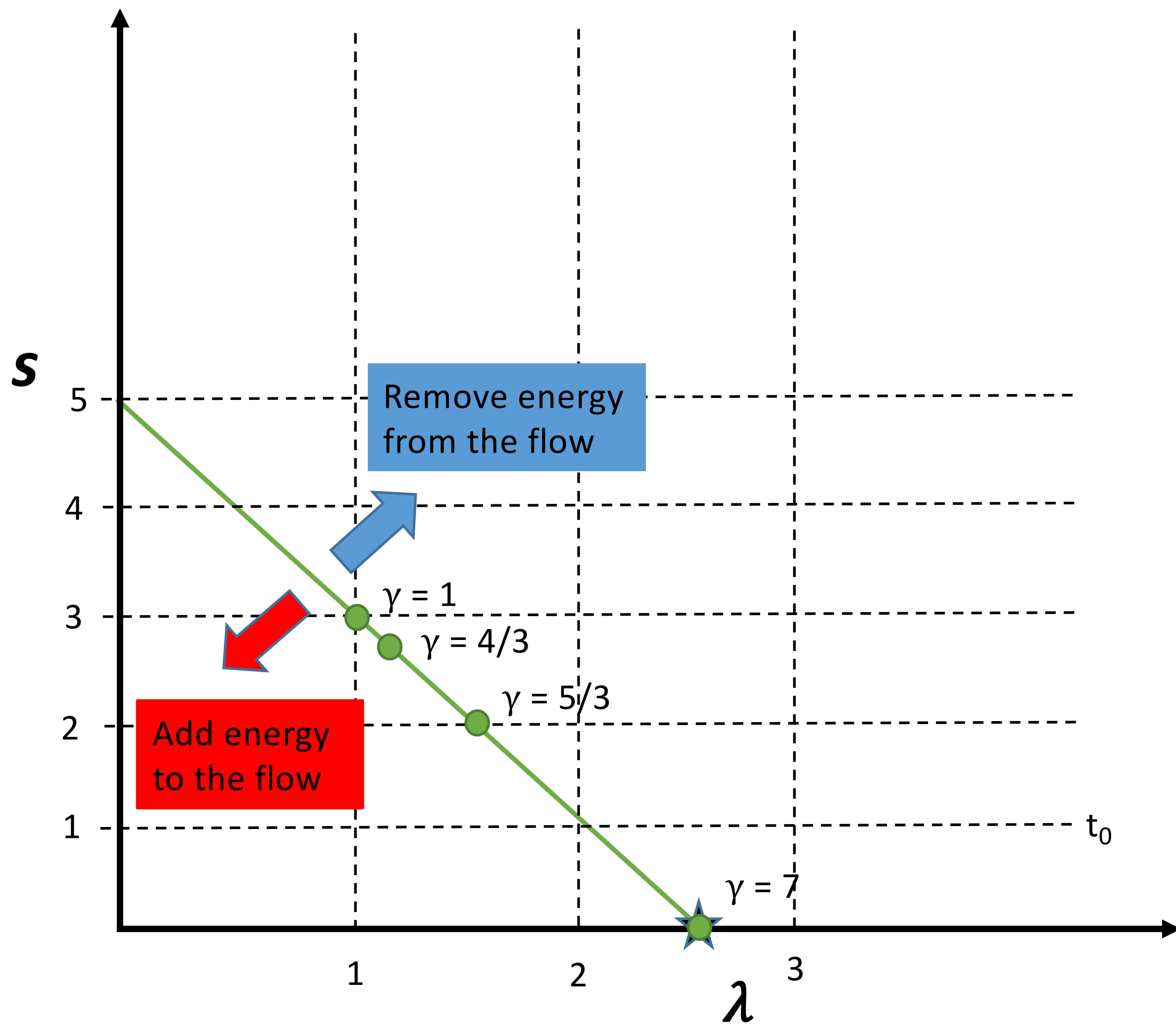
Sometimes, yes---but usually not so much.

$$u = \frac{r}{t}U(\zeta) \qquad \rho = \frac{1}{r^s}\Omega(\zeta) \qquad \frac{p}{\rho} \propto \frac{r^2}{t^2}$$

$$\zeta = \frac{r}{t^{1/\lambda}}$$

They provide three nonlinear coupled ODE's for U, Ω, and Π. Boundary conditions yield λ. They are called the **blast-wave** equations.


time
radius
R0
t0
$R = R_0(t)$
$\zeta = \zeta_0 \left(\frac{t}{t_0}\right)^{1/\lambda}$
$u = \frac{r}{t} U(\zeta) = \frac{dr}{dt}$
$\zeta = \frac{r}{t^{1/\lambda}}$
$R_{shock} = R_{shock_0} \left(\frac{t}{t_0}\right)^{1/\lambda}$
$\rho_\infty \propto \left(\frac{1}{r}\right)^s$



1.3. ÉCOULEMENTS COMPRESSIBLES 23

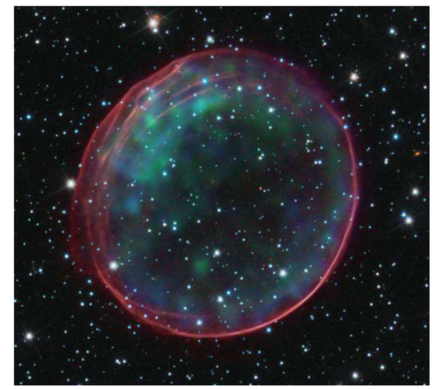


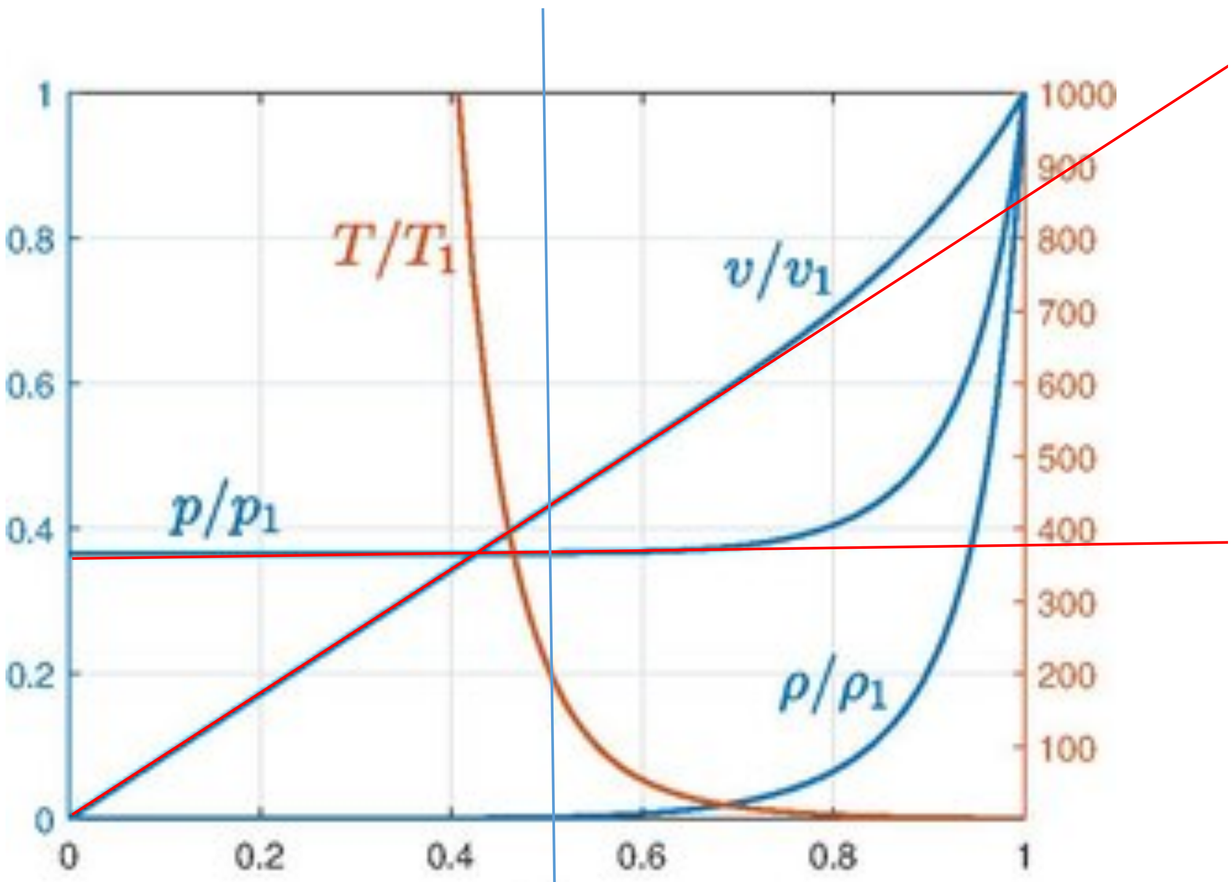
Figure 1.5: Onde de détonation produite par une supernova ayant pêté il y a plus de 400 ans (SNR 0509-67.5). Cette imagine combine des données optiques (rouge) et en rayon-X (vert-bleu). L'émission optique est associée au chauffage du milieu interstellaire par le passage de l'onde de choc. Image de la NASA, en domaine public.

et sa vitesse d'expansion est donnée par

★ $$v_c(t) = \frac{\mathrm{d}r_c}{\mathrm{d}t} = \frac{2\xi_0}{5}\left(\frac{E}{\varrho_1 t^3}\right)^{1/5} . \quad \gamma = 5/3 \qquad (1.90)$$

$$R_{shock} = R_{shock_0}\left(\frac{t}{t_0}\right)^{1/\lambda}$$

$$\rho_\infty \propto \left(\frac{1}{r}\right)^{s}$$



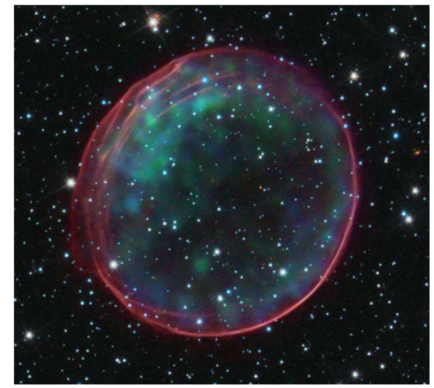
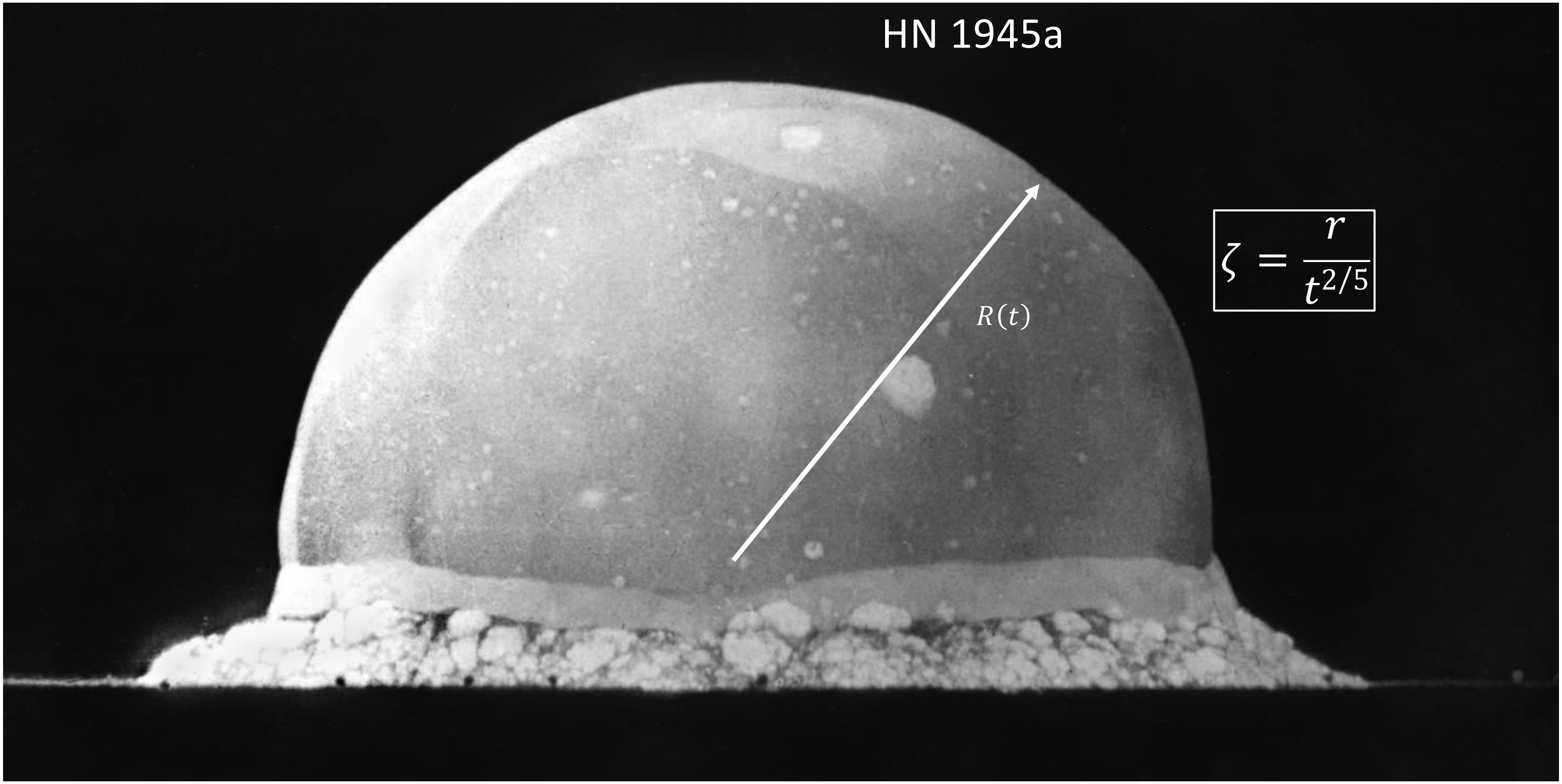


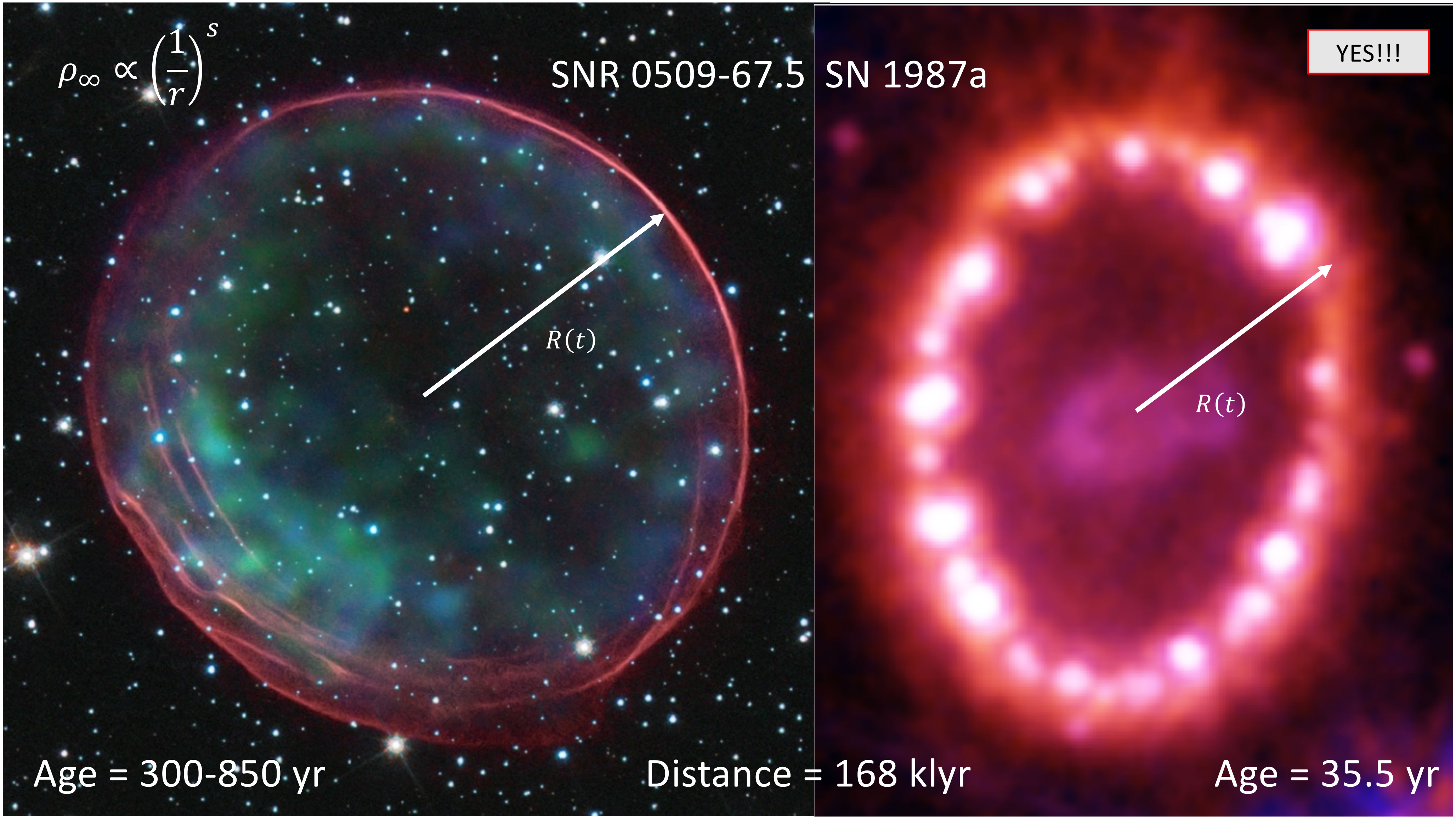
Figure 1.5: Onde de détonation produite par une supernova ayant pêté il y a plus de 400 ans (SNR 0509-67.5). Cette imagine combine des données optiques (rouge) et en rayon-X (vert-bleu). L'émission optique est associée au chauffage du milieu interstellaire par le passage de l'onde de choc. Image de la NASA, en domaine public.

et sa vitesse d'expansion est donnée par

$$v_c(t) = \frac{\mathrm{d}r_c}{\mathrm{d}t} = \frac{2\xi_0}{5}\left(\frac{E}{\varrho_1 t^3}\right)^{1/5} . \qquad (1.90)$$

$$R_{shock} = R_{shock_0}\left(\frac{t}{t_0}\right)^{2/5}$$


HN 1945a
$R(t)$
$\zeta = \frac{r}{t^{2/5}}$



$\rho_\infty \propto \left(\frac{1}{r}\right)^s$
SNR 0509-67.5
SN 1987a
YES!!!
$R(t)$
$R(t)$
Age = 300-850 yr
Distance = 168 klyr
Age = 35.5 yr

- It is ***cosmically unlikely*** that the density outside of a supernova cares about the ratio of specific heats---which has to be 4/3 at least early on---giving $r^{-17/7}$ (!!!)

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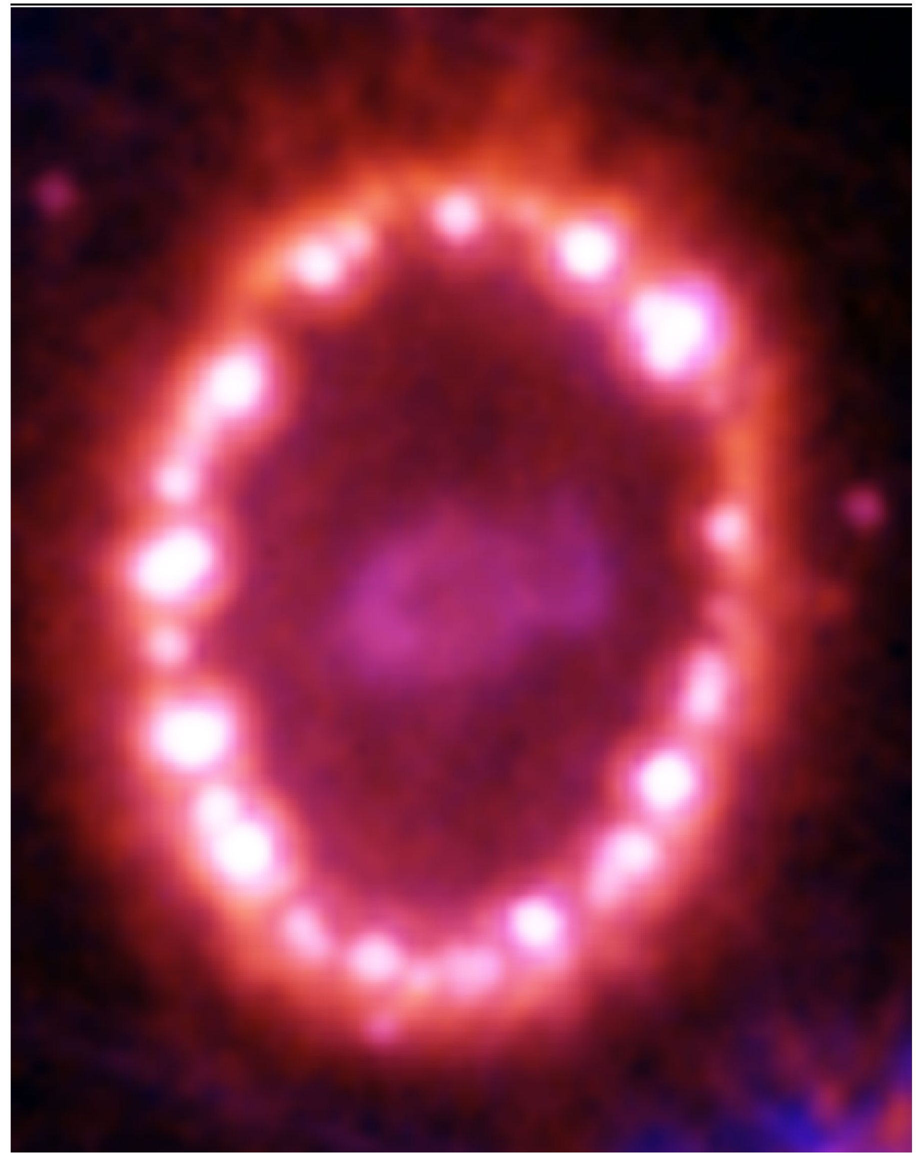
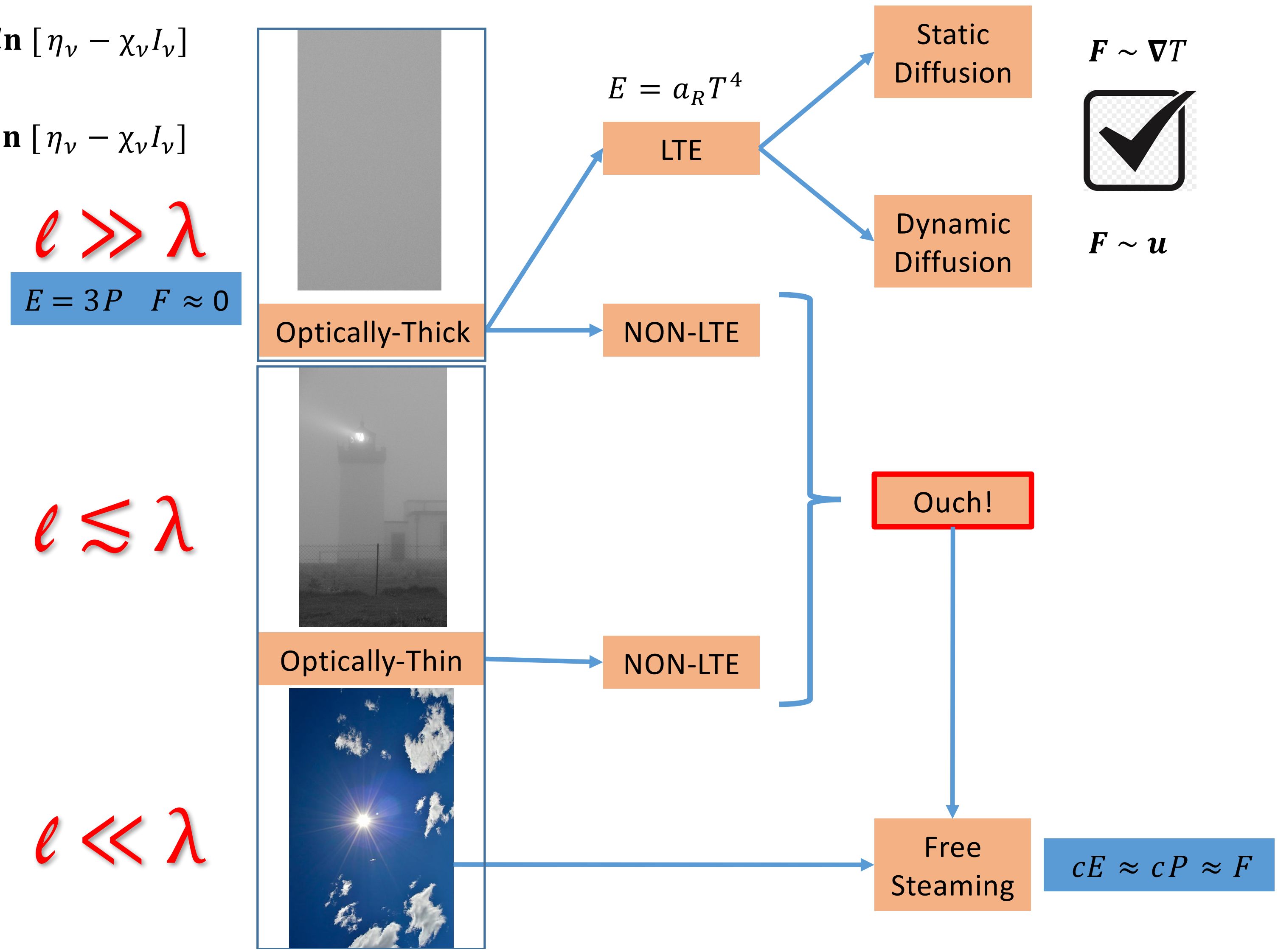


$$\frac{\partial E}{\partial t} + \boldsymbol{\nabla}\cdot \boldsymbol{F} = \int_0^\infty d\nu \int d\mathbf{n}\,[\eta_\nu - \chi_\nu I_\nu]$$

$$\frac{1}{c^2}\frac{\partial \boldsymbol{F}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbb{P} = \frac{1}{c}\int_0^\infty d\nu\int d\mathbf{n}\,\mathbf{n}\,[\eta_\nu-\chi_\nu I_\nu]$$

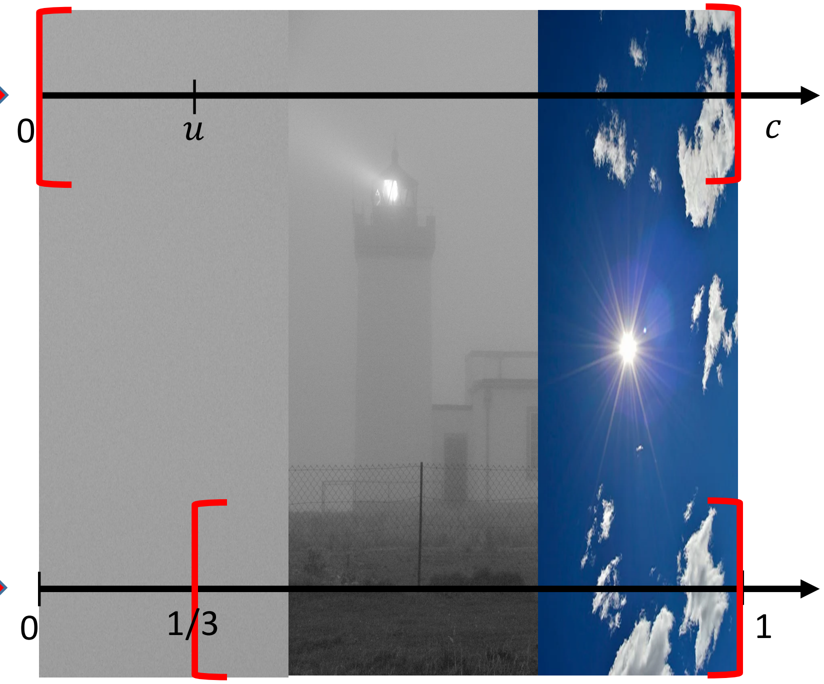


Summary

Summary

$$\frac{\boldsymbol{F}_\nu = \oint \mathrm{d}\mathbf{n}\, \mathbf{n}\, I_\nu}{E_\nu = \frac{1}{c}\oint \mathrm{d}\mathbf{n}\, I_\nu}$$

$$\frac{\mathbb{P}_\nu = \frac{1}{c}\oint \mathrm{d}\mathbf{n}\, \mathbf{n}\mathbf{n}\, I_\nu}{E_\nu = \frac{1}{c}\oint \mathrm{d}\mathbf{n}\, I_\nu}$$



Eddington Factor

Diffusion Limit

Free Streaming

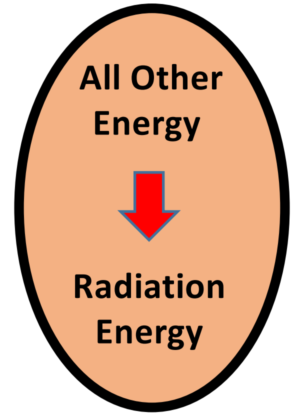
$$I_\nu \approx \frac{c}{4\pi} E_\nu + \frac{3}{4\pi} \mathbf{n} \cdot \boldsymbol{F}_\nu$$

Ouch!

$$I_\nu \approx c E_\nu \delta(\mu - 1)$$

The Radiation Field

In the fixed inertial frame of the star.



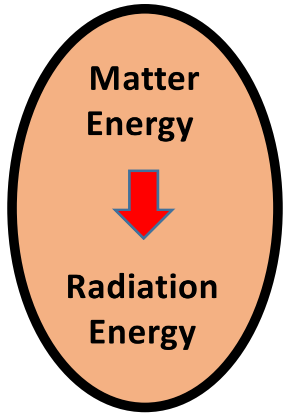
$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I_\nu = \eta_\nu - \chi_\nu I_\nu$$

$$\frac{\partial E}{\partial t} + \nabla\cdot\boldsymbol{F} = \int_0^\infty d\nu \int d\mathbf{n}\,[\eta_\nu - \chi_\nu I_\nu]$$

$$\frac{1}{c^2}\frac{\partial \boldsymbol{F}}{\partial t} + \nabla\cdot\mathbb{P} = \frac{1}{c}\int_0^\infty d\nu \int d\mathbf{n}\,\mathbf{n}[\eta_\nu - \chi_\nu I_\nu]$$

Best evaluated in the comoving frame.

In the non-inertial comoving frame



The Radiation Field

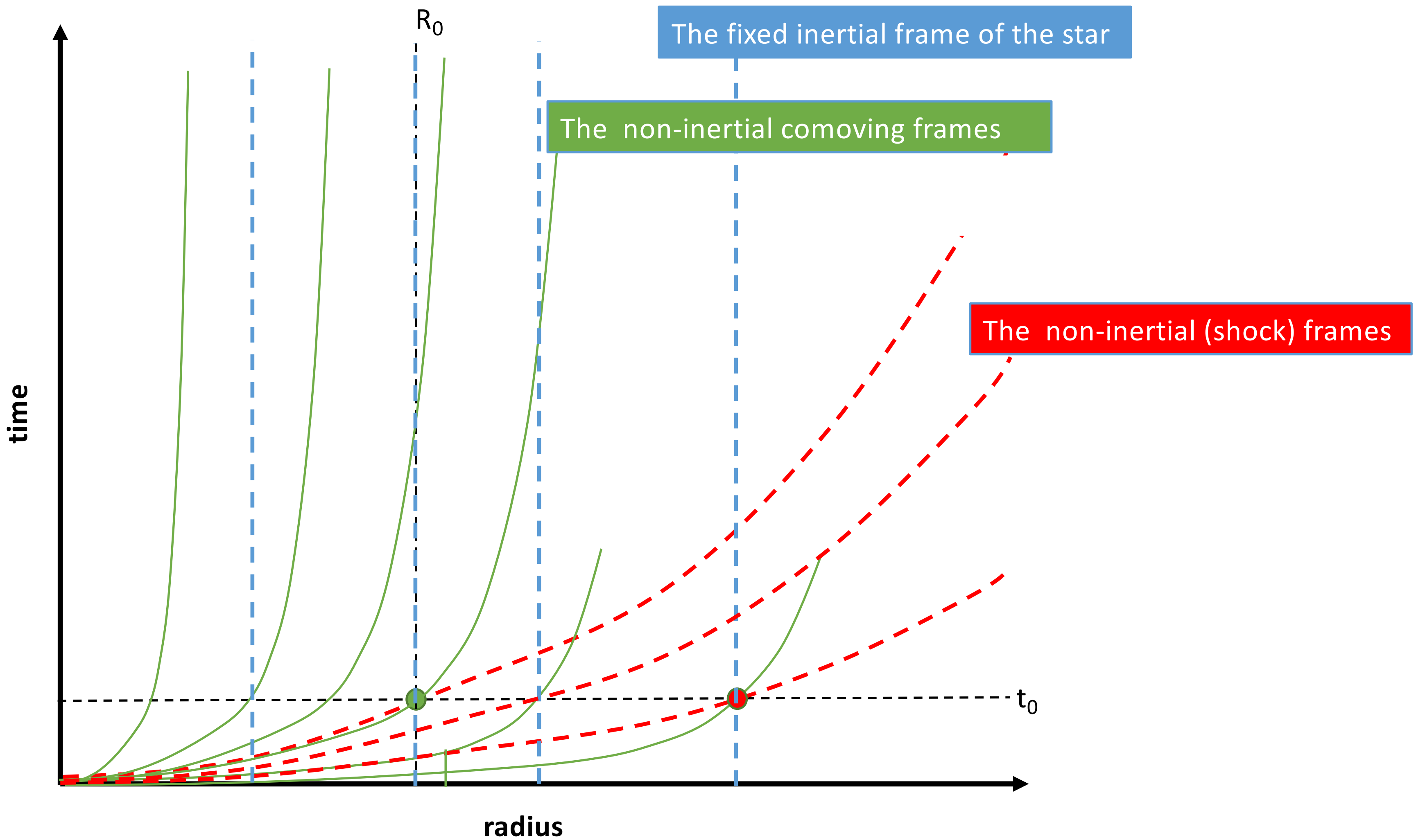
$$\frac{1}{c}\frac{\partial I'_{\nu'}}{\partial t'} + \cdots + \mathbf{n}' \cdot \nabla' I'_{\nu'} = \eta'_{\nu'} - \chi'_{\nu'} I'_{\nu'}$$

Gray Approximation + LTE Approximation + Isotropic Scattering

$$\frac{\partial E'}{\partial t'} + \nabla' \cdot \boldsymbol{F}' = \kappa'[4\sigma_R T'^4 - cE'] +$$

$$\frac{1}{c^2}\frac{\partial \boldsymbol{F}'}{\partial t'} + \nabla' \cdot \mathbb{P}' = -\frac{\kappa'}{c}\boldsymbol{F}' - \frac{\sigma'}{c}\boldsymbol{F}' +$$

Non-inertial terms now live here.



R0
The fixed inertial frame of the star
The non-inertial comoving frames
The non-inertial (shock) frames
time
t0
radius

The Radiation Field

Mixed frame formalism

Matter Energy

↓

Radiation Energy

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I_\nu = \eta_\nu - \chi_\nu I_\nu$$

$$-G^\alpha = \frac{1}{c}\int_0^\infty d\nu \int d\mathbf{n}\, n^\alpha[\eta_\nu - \chi_\nu I_\nu]$$

$\mathcal{L}^{-1}$

$$\{E, F, \mathbb{P}\} \rightarrow \{E', F', \mathbb{P}'\} \longleftrightarrow \{E', F', \mathbb{P}'\}$$

$$\frac{\partial E}{\partial t} + \nabla\cdot\boldsymbol{F} = \int_0^\infty d\nu \int d\mathbf{n}\,[\eta_\nu - \chi_\nu I_\nu]$$

$$\frac{1}{c^2}\frac{\partial \boldsymbol{F}}{\partial t} + \nabla\cdot\mathbb{P} = \frac{1}{c}\int_0^\infty d\nu \int d\mathbf{n}\,\mathbf{n}[\eta_\nu - \chi_\nu I_\nu]$$

$\mathcal{L}$

$$\{G^{0\prime}, \boldsymbol{G}'\} \rightarrow \{G^0, \boldsymbol{G}\}$$

The resulting equations contain terms proportional to powers of u ***and*** its derivatives with respect to r and t.


E=mc²
PV=nRT
A=13%
B=25%
C=?

$$\frac{\partial}{\partial t}\rho + \boldsymbol{\nabla}\cdot\rho\boldsymbol{u} = 0$$

$$\frac{\partial}{\partial t}\rho\boldsymbol{u} + \frac{1}{3}\boldsymbol{\nabla}\cdot\left(a_R T^4 + 3\rho\boldsymbol{u}\boldsymbol{u}\right) = 0$$

$$\left[\frac{\partial}{\partial t} + \frac{4}{3}\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla} - \boldsymbol{\nabla}\cdot\frac{3c}{\chi}\boldsymbol{\nabla}\right] a_R T^4 = \rho T\dot{s}$$

$$u = \varepsilon\frac{r}{t}$$

$$u = \frac{r}{t}U(\zeta) \qquad \zeta = \frac{r}{t^{1/\lambda}}$$

$$u = \frac{r}{\Phi}\frac{\mathrm{d}\Phi}{\mathrm{d}t}U(\zeta) \qquad \zeta = \frac{r}{\Phi}$$

- We have omitted gravity (needed only for Nanonovas---stay tuned).
- Neglected the internal energy density of the matter compared to the radiation energy density (should be OK for a while…).
- Omitted terms of order u/c (probably OK after the shock breaks through the stellar photosphere…)
- Assumed the photon mean-free-path is much smaller than any relevant hydrodynamic scale (OK almost everywhere except within 10 or so optical depths of the photosphere…)
- Frequency-integrated moments (not so OK…)
- Assumed LTE (not so OK…)
- Neglected neutrinos (---yes ***NEUTRINOS***---OK after the shock breaks through the stellar photosphere…

$$\left\{ \begin{aligned} &\frac{\partial}{\partial t}\rho + \boldsymbol{\nabla}\cdot\rho\boldsymbol{u} = 0 \\ &\frac{\partial}{\partial t}\rho\boldsymbol{u} + \frac{1}{3}\boldsymbol{\nabla}\cdot(a_R T^4 + 3\rho\boldsymbol{u}\boldsymbol{u}) = 0 \\ &\left[\frac{\partial}{\partial t} + \frac{4}{3}\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla} - \boldsymbol{\nabla}\cdot\frac{3c}{\chi}\boldsymbol{\nabla}\right]a_R T^4 = \rho T\dot{s} \end{aligned} \right.$$

With r held constant!

$$a_R T^4 = \frac{\varphi(t)}{\Phi^4}\Pi(\zeta)$$

$$u = \frac{r}{\Phi}\frac{\mathrm{d}\Phi}{\mathrm{d}t}U(\zeta)$$

$$\zeta = \frac{r}{\Phi}$$

$$\rho = \frac{1}{\Phi^3}\Omega(\zeta)$$

$$\begin{cases} \dfrac{\partial}{\partial t}\rho + \boldsymbol{\nabla}\cdot\rho\boldsymbol{u} = 0 \\[2ex] \dfrac{\partial}{\partial t}\rho\boldsymbol{u} + \dfrac{1}{3}\boldsymbol{\nabla}\cdot\left(a_R T^4 + 3\rho\boldsymbol{u}\boldsymbol{u}\right) = 0 \\[2ex] \left[\dfrac{\partial}{\partial t} - \boldsymbol{\nabla}\cdot\dfrac{3c}{\chi}\boldsymbol{\nabla}\right]\Phi^4 a_R T^4 = \Phi^4 \rho T \dot{s} \end{cases}$$

They provide three nonlinear coupled ODE's for U, Ω, and Π. Boundary conditions yield $\Phi(\mathrm{t})$, $\varphi(\mathrm{t})$.

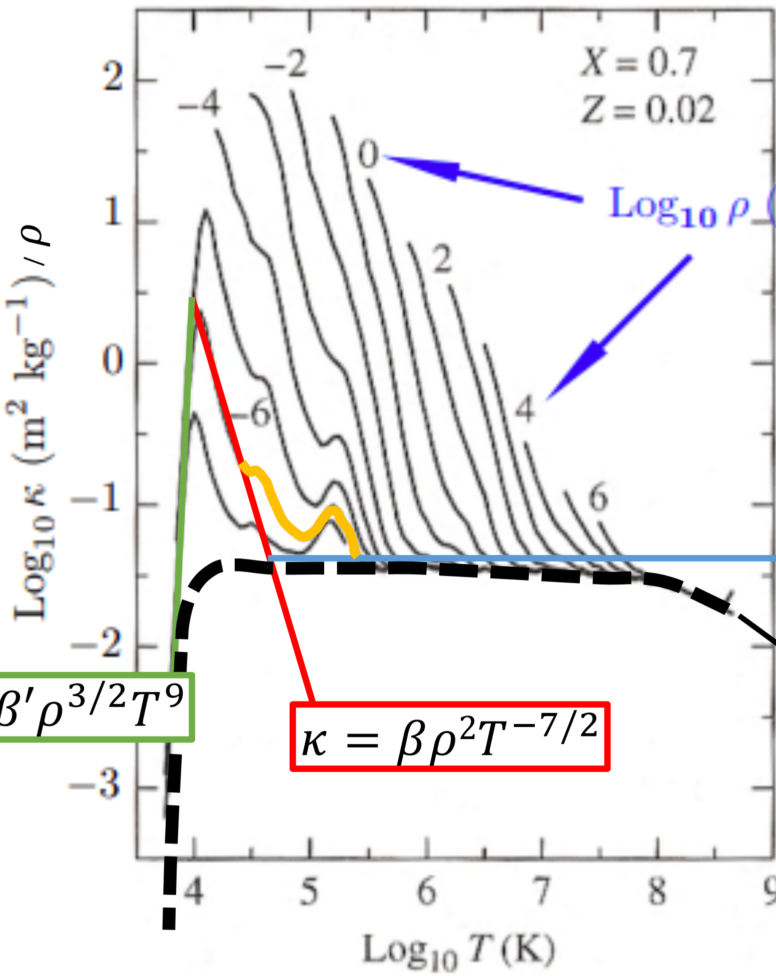
With ζ held constant!

$$\Phi^4 a_R T^4 = \varphi(t)\Pi(\zeta)$$

$$u = \frac{r}{\Phi}\frac{\mathrm{d}\Phi}{\mathrm{d}t}U(\zeta)$$

$$\zeta = \frac{r}{\Phi}$$

$$\rho = \frac{1}{\Phi^3}\Omega(\zeta)$$



H- opacity

Bound-free and Free-free

Scattering off free electrons

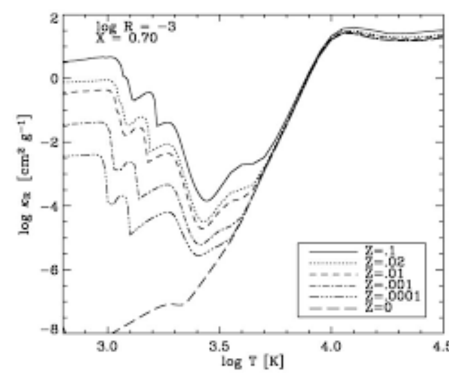
$$\gamma = \frac{8\pi}{3m_H}\left(\frac{e^2}{4\pi\varepsilon_0 mc^2}\right)^2$$

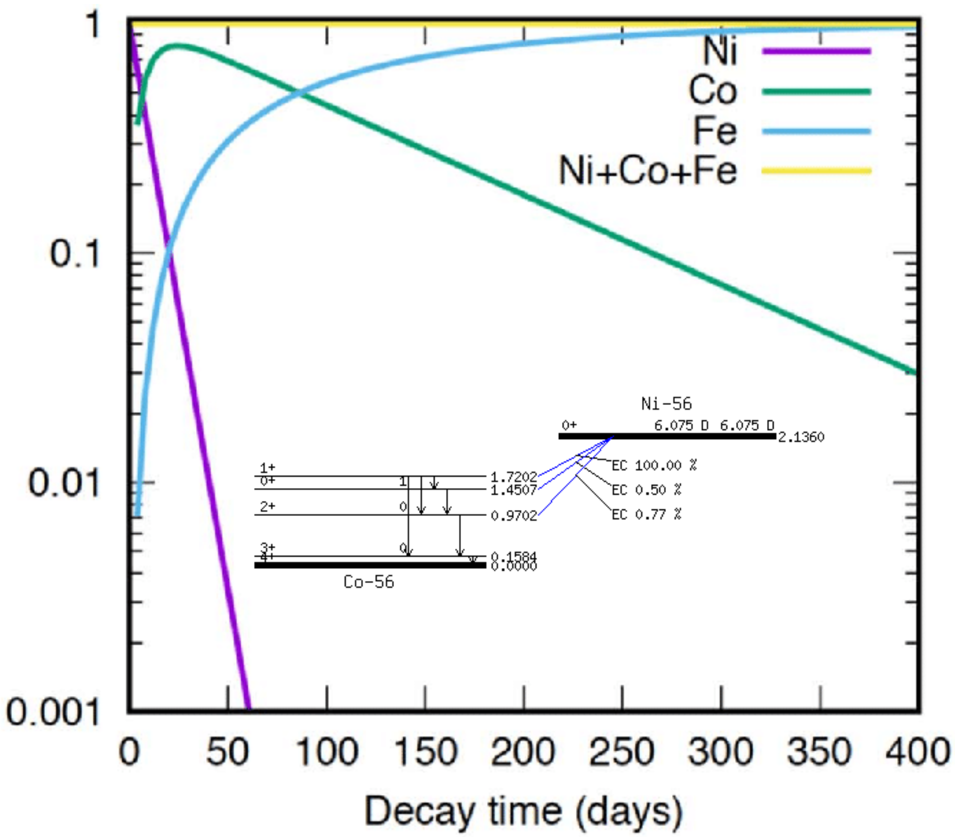
$$\frac{\chi}{\rho} \approx 4 - 0.04 \text{ m}^2/\text{kg}$$

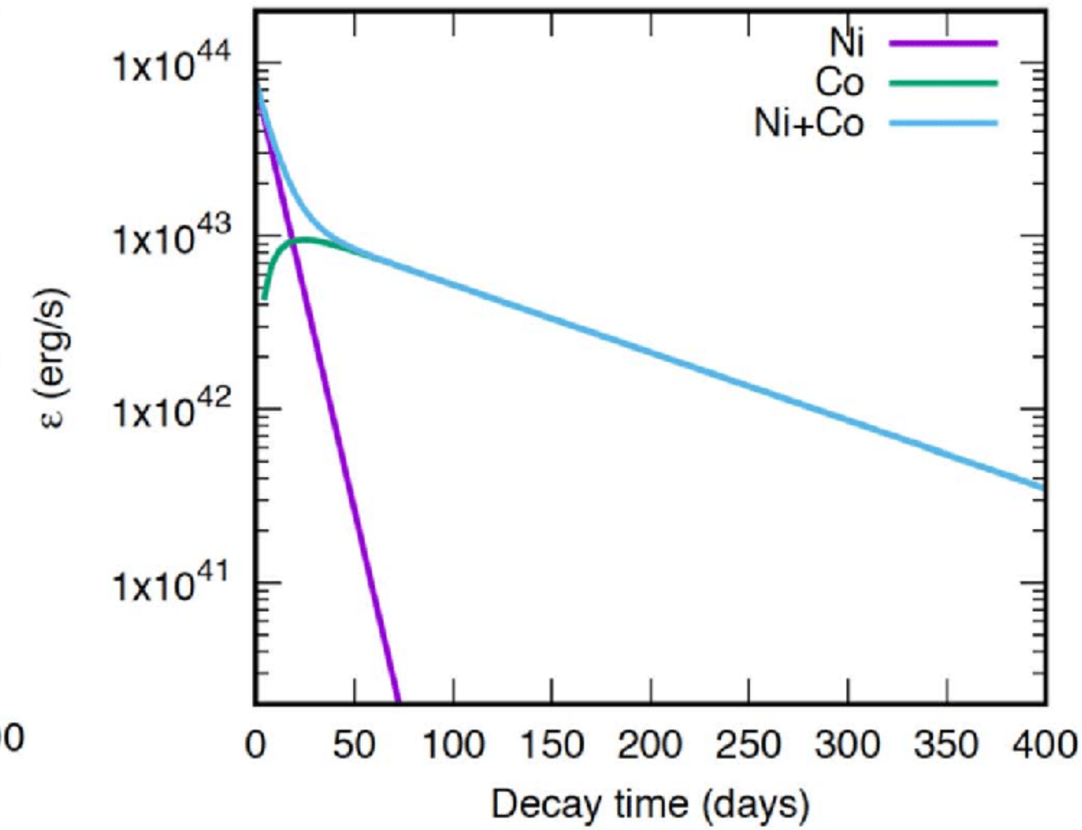
$\kappa = \beta' \rho^{3/2} T^9$

$\kappa = \beta \rho^2 T^{-7/2}$

Relativistic effects





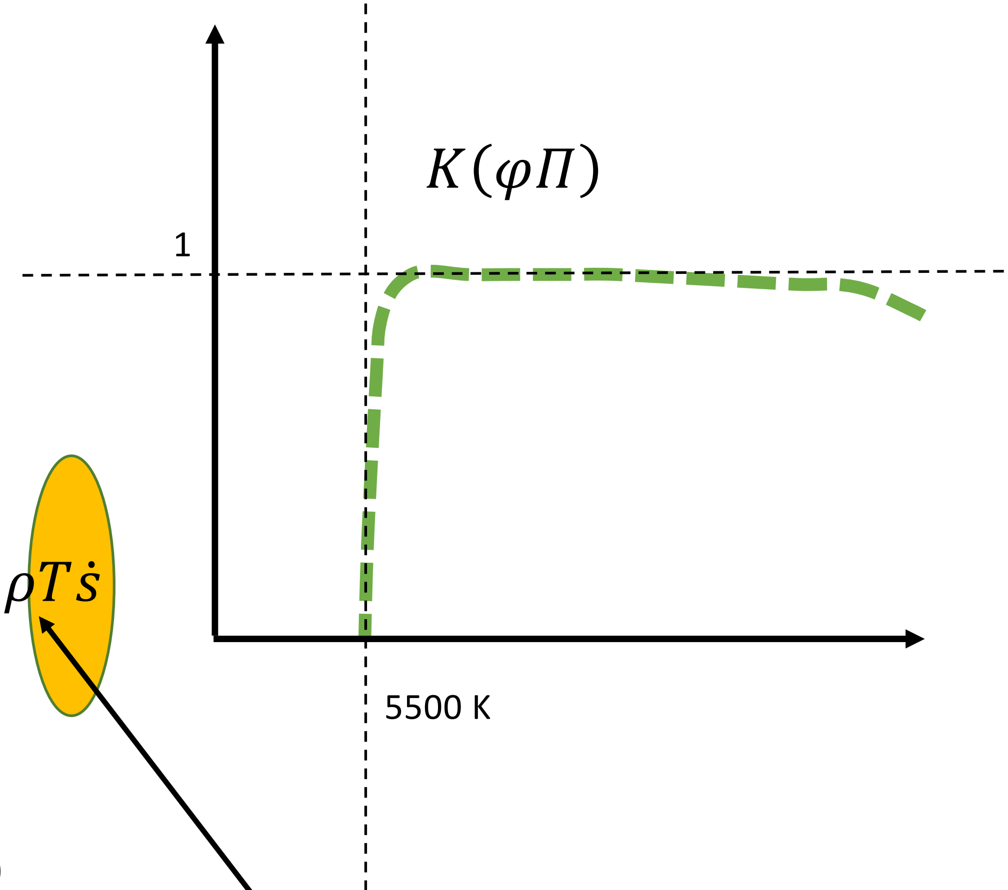


$N_1 = 1 \quad \nu_0 = 0$

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = -\nu_i N_i + \nu_{i-1} N_{i-1}$$

$$\nu(t) \equiv T\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\mathrm{d}Q}{\mathrm{d}t} = \sum_i \varepsilon_i \nu_i N_i$$

$$\zeta = \frac{r}{\Phi} \qquad \Phi^4 a_R T^4 = \varphi(t)\Pi(\zeta)$$



$$\left[\frac{\partial}{\partial t} - \boldsymbol{\nabla}\cdot\frac{3c}{\chi}\boldsymbol{\nabla}\right]\Phi^4 a_R T^4 = \Phi^4 \rho T\dot{s}$$

With ζ held constant!

$$\rho = \frac{1}{\Phi^3}\Omega(\zeta)$$

$$\varkappa = \varkappa_0 K(\varphi\Pi)$$

$$\nu(t) \equiv T\dot{s} = \sum_i \varepsilon_i \nu_i N_i$$

❶ ❷ ❸

$$\frac{\dot{\varphi}}{\varphi}\frac{1}{\Phi}-\frac{3c}{\varkappa_0}\frac{1}{\Pi\zeta^2}\frac{\mathrm{d}}{\mathrm{d}\zeta}\left(\frac{\zeta^2}{\Omega K}\frac{\mathrm{d}\Pi}{\mathrm{d}\zeta}\right)=\frac{\Omega}{\Pi}\frac{\nu}{\varphi}$$

Rate of change of the energy density in the radiation field---which depends only upon ***time***.

The divergence of the radiative flux---which depends only upon the ***spatial similarity variable***.

Heating (which ends up as photons) due to radioactive decay.

This depends on ***both*** space and time, unfortunately.

1. Balance any ***<u>two</u>*** terms and see if the third term is generally negligible
2. Integrate $4\pi r^2$ dr out to $R_{shock}(t)$ to remove the spatial dependence away
3. Give up on the similarity assumption--try $a_R T^4 = \Psi(r,t)/\Phi^4$ or:

$$\left[\frac{\partial}{\partial t}-\nabla\cdot\frac{3c}{\rho\varkappa}\nabla\right]\Phi^4 a_R T^4=\Phi^4\rho\nu$$

$$\frac{1}{\Phi}\frac{\partial}{\partial t}=\frac{\partial}{\partial\tau}$$

$$\nu(t)=\sum_i \varepsilon_i \nu_i N_i$$

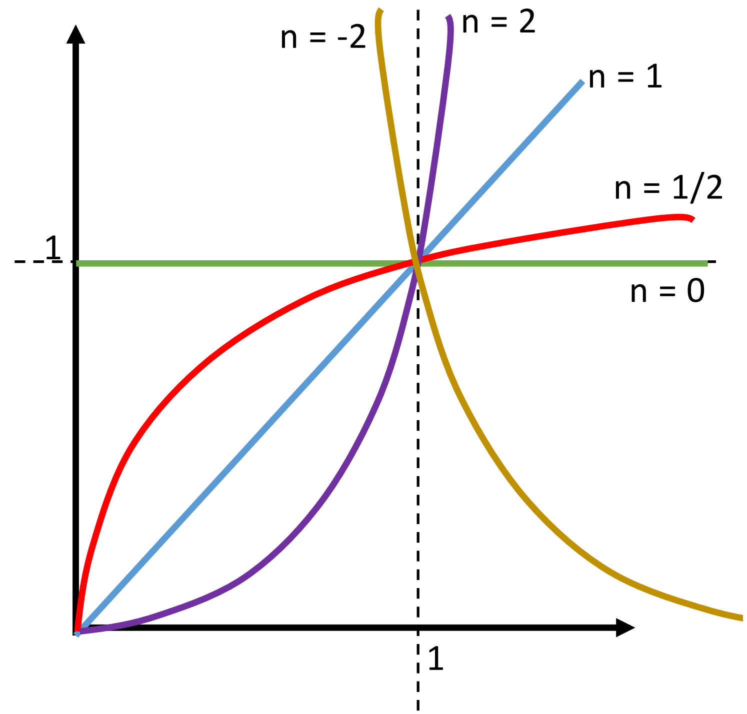
$$\zeta=\frac{r}{\Phi}$$

$$K(\varphi\Pi)$$

❷ ❶

$$\frac{1}{\zeta^2}\frac{d}{d\zeta}\left(\frac{\zeta^2}{\Omega}\frac{d\Pi}{d\zeta}\right) = -\beta^2\Pi - \Omega$$

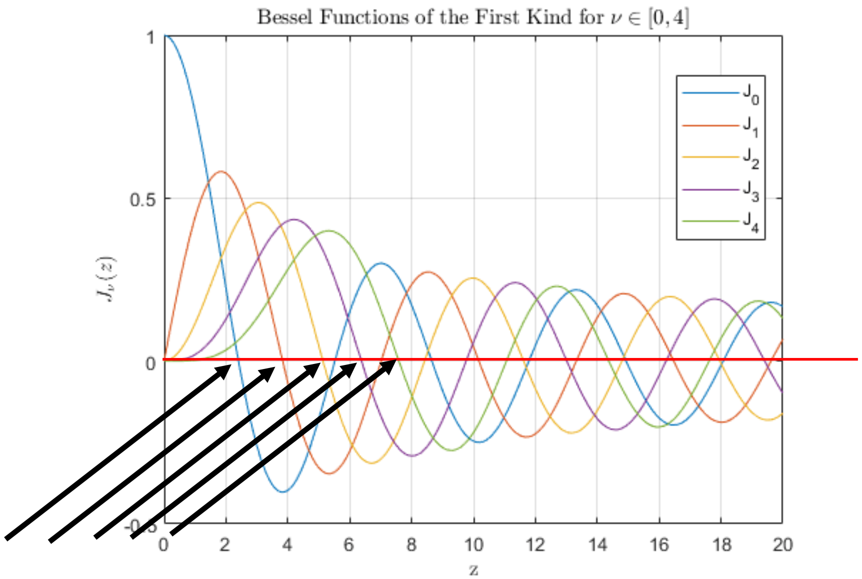
$$\Omega = \zeta^n$$



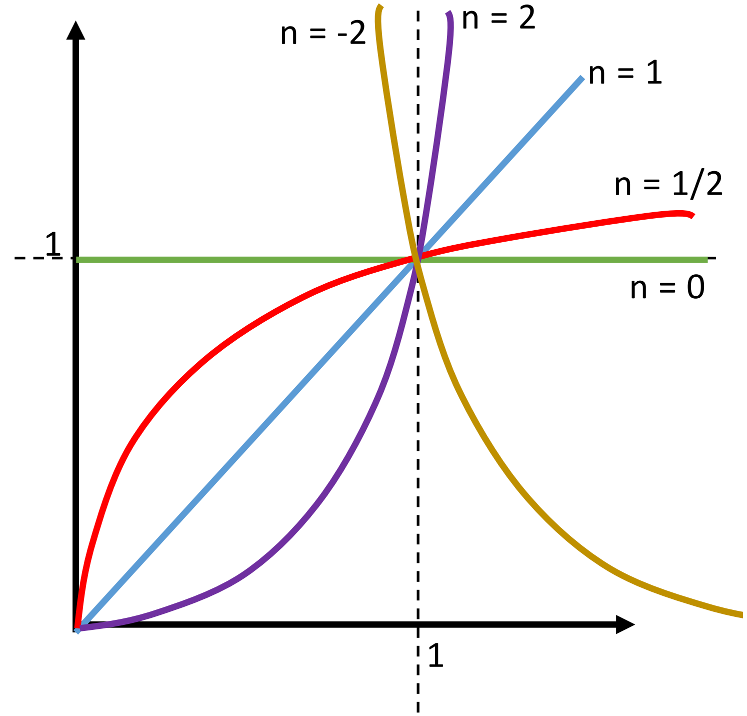
$$\frac{\dot{\varphi}}{\varphi}\frac{1}{\Phi} - \frac{3c}{\varkappa_0}\frac{1}{\Pi\zeta^2}\frac{d}{d\zeta}\left(\frac{\zeta^2}{\Omega K}\frac{d\Pi}{d\zeta}\right) = \frac{\Omega}{\Pi}\frac{\nu}{\varphi}$$

$$\zeta^2\frac{d^2\Pi}{d\zeta^2} + (2-n)\zeta\frac{d\Pi}{d\zeta} + \beta^2\zeta^{n+2}\Pi = -\zeta^{2+2n}$$

$$\zeta^{\frac{n-1}{2}} J_{\pm\mu}\left(\frac{2\beta}{n+2}\zeta^{1+\frac{n}{2}}\right) \qquad \mu = \left|\frac{n-1}{n+2}\right|$$



$$\Omega = \zeta^n$$



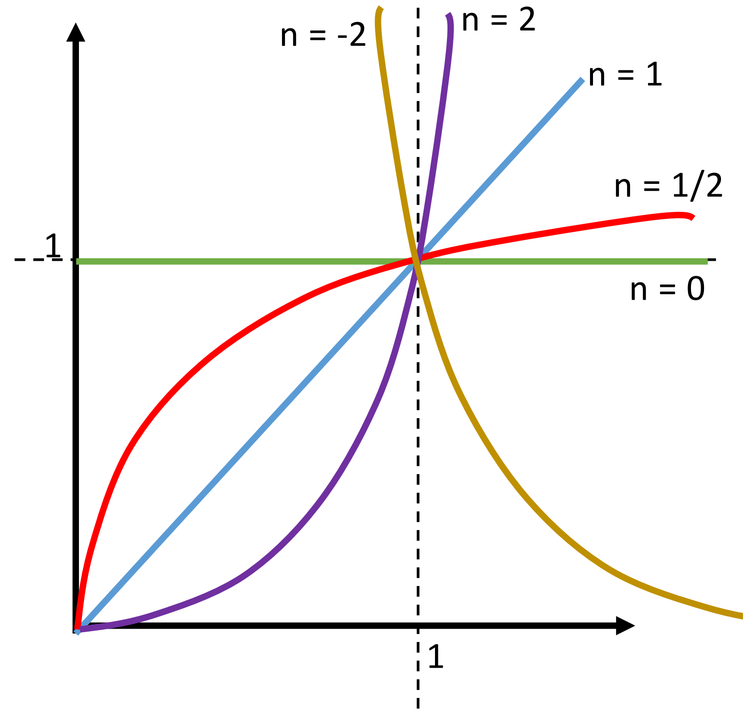
$$\zeta^2 \frac{\mathrm{d}^2 \Pi}{\mathrm{d}\zeta^2} + (2-n)\zeta \frac{\mathrm{d}\Pi}{\mathrm{d}\zeta} + \beta^2 \zeta^{n+2} \Pi = -\zeta^{2+2n}$$

$$\zeta^{\frac{n-1}{2}} J_{\pm\mu}\left(\frac{2\beta}{n+2}\zeta^{1+\frac{n}{2}}\right) \qquad \mu = \left|\frac{n-1}{n+2}\right|$$

❷ ❸

$$\frac{1}{\zeta^2}\frac{\mathrm{d}}{\mathrm{d}\zeta}\left(\frac{\zeta^2}{\Omega}\frac{\mathrm{d}\Pi}{\mathrm{d}\zeta}\right) =- \beta^2\Pi - \Omega$$

$$\Omega = \zeta^n$$



$$\frac{\dot{\varphi}}{\varphi}\frac{1}{\Phi} - \frac{3c}{\varkappa_0}\frac{1}{\Pi\zeta^2}\frac{\mathrm{d}}{\mathrm{d}\zeta}\left(\frac{\zeta^2}{\Omega K}\frac{\mathrm{d}\Pi}{\mathrm{d}\zeta}\right) = \frac{\Omega}{\Pi}\frac{\nu}{\varphi}$$

$$\zeta^2\frac{\mathrm{d}^2\Pi}{\mathrm{d}\zeta^2} + (2-n)\zeta\frac{\mathrm{d}\Pi}{\mathrm{d}\zeta} + \beta^2\zeta^{n+2}\Pi = -\zeta^{2+2n}$$

$$\Pi_0 + \Pi_1\zeta^{n-2} - \frac{\zeta^{2+2n}}{2(n+1)(n+3)}$$

$$\frac{\partial}{\partial t}\rho + \boldsymbol{\nabla}\cdot\rho\boldsymbol{u} = 0$$

$$\frac{\partial}{\partial t}\rho\boldsymbol{u} + \frac{1}{3}\boldsymbol{\nabla}\cdot(a_R T^4 + 3\rho\boldsymbol{u}\boldsymbol{u}) = 0$$

$$\left[\frac{\partial}{\partial t} - \boldsymbol{\nabla}\cdot\frac{3c}{\chi}\boldsymbol{\nabla}\right]\Phi^4 a_R T^4 = \Phi^4\rho T\dot{s}$$

With ζ held constant!

$$\Phi^4 a_R T^4 = \varphi(t)\Pi(\zeta) = \Psi(\zeta,t)$$

$$u = \frac{r}{\Phi}\frac{\mathrm{d}\Phi}{\mathrm{d}t}U(\zeta)$$

$$\zeta = \frac{r}{\Phi}$$

$$\rho = \frac{1}{\Phi^3}\Omega(\zeta)$$

$$R_{shock} = R_{shock_0}\left(\frac{t}{t_0}\right)^{1/\lambda}$$

Astrophysical Radiation Magnetohydrodynamics

Being an Opera in Four Acts

CURTAIN
CALL

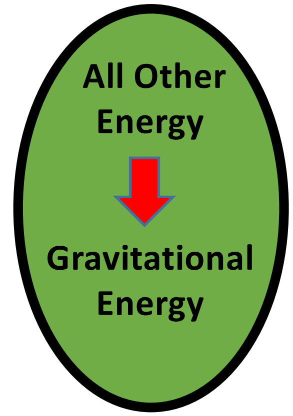
Problem 1

The Gravitational Field Problem

$$\nabla^2\Phi = 4\pi G\rho$$

$$\frac{\partial}{\partial t}\rho + \nabla\cdot\rho\boldsymbol{u} = 0$$

Note: None of the gravity is provided by the material we are keeping track of via the continuity equation.

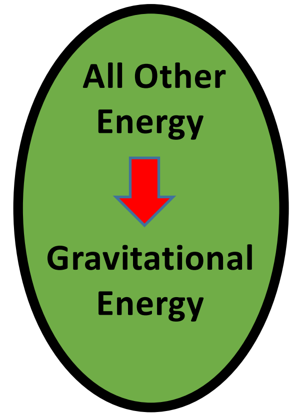


$$\frac{\partial}{\partial t}\frac{1}{2}\rho\Phi + \nabla\cdot\left(\rho\Phi\boldsymbol{u} + \boldsymbol{G}\right) = \rho\boldsymbol{u}\cdot\nabla\Phi$$

$$\nabla\cdot\mathbb{G} = \rho\nabla\Phi$$

Problem 1 | **Solved!**

The Gravitational Field Problem



$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \pm \frac{8\pi G}{c^4} T_{\mu\nu}$$

Note: The divergence of <u>both</u> sides of this equation vanish identically!

Half the world prefers + and the other half − ...

This term has contributions from the matter, the electromagnetic fields, and the radiation field. Its divergence, set equal to zero, gives the relativistic generalization of our (summed) RMHD equations! The gravitational energy/momentum exchange and conservations are now described by the metric coefficients and the Christoffel symbols associated with the covariant derivatives!

Problem 2 | **Solved…**

The Closure (No Free Lunch) Problem

$$\frac{p}{\rho} = (\gamma - 1)e + \cdots$$

$$\mathbb{P} = \frac{1}{c}\int_0^\infty d\nu \int d\mathbf{n}\,\mathbf{n}\mathbf{n}\,I_\nu$$

$$\eta_\nu = \chi_\nu B_\nu + \cdots$$

(Ionization/Recombination ?)
(Dissociation/Recombination ?)
(Pair Production/Annihilation ?)
(Nuclear Reactions ?)

$$\Omega = -k_B T \log Z\,(V, T, \boldsymbol{\mu})$$

Equilibrium statistical mechanics

$$\boldsymbol{J} = \sigma \boldsymbol{E} + \cdots$$

$$\rho_e = 0 + \cdots$$

$$\chi_\nu = 1/\lambda$$

(Thermal conductivity ?)
(Viscous stresses ?)
(Chemical diffusion ?)
(Anisotropy ?)

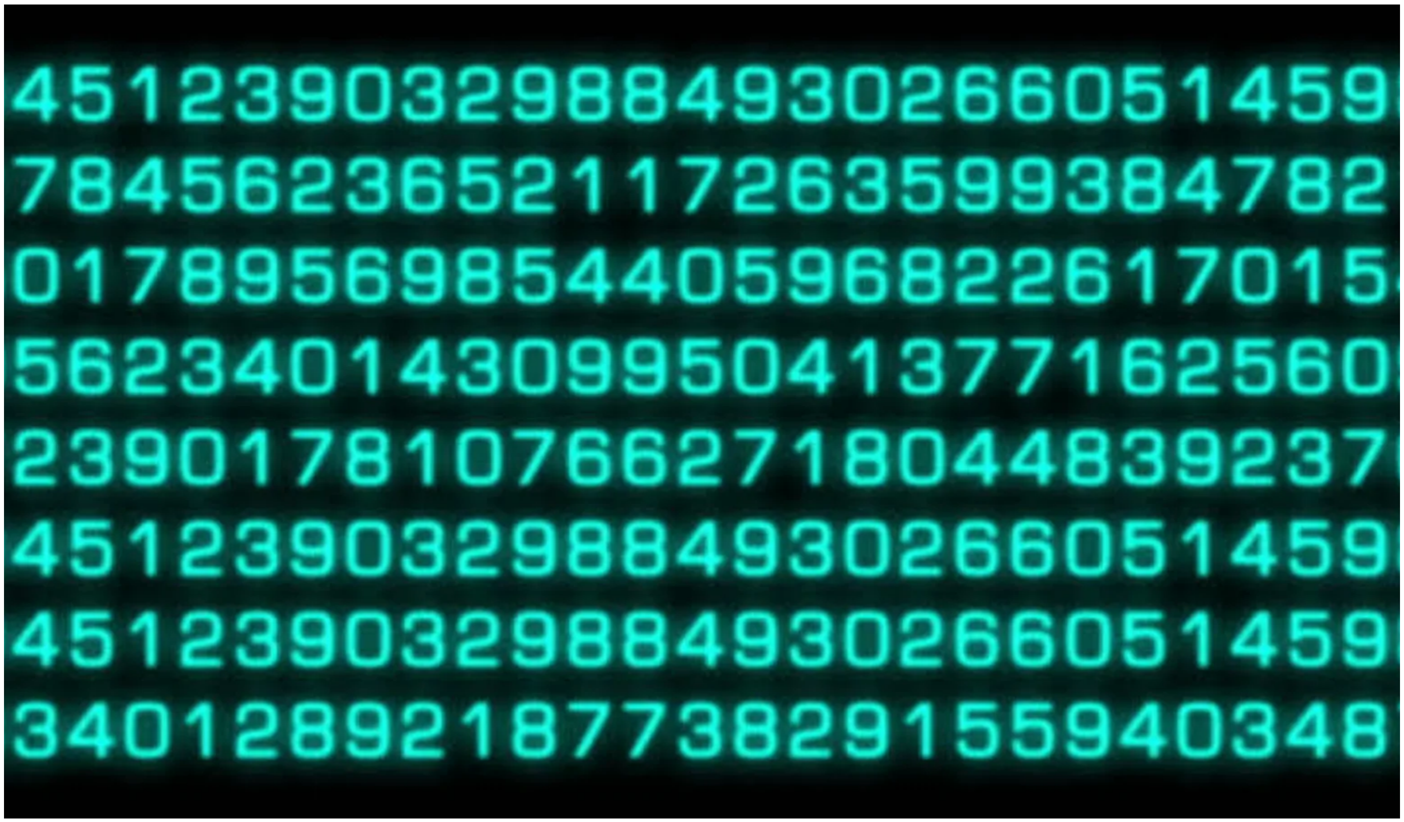
Non-equilibrium transport

$$\frac{\partial I_\nu}{\partial t} + c\boldsymbol{n}\cdot\frac{\partial I_\nu}{\partial \boldsymbol{x}} = c\eta_\nu - c\chi_\nu I_\nu$$

$$\frac{\partial \psi}{\partial t} + \frac{1}{m}\boldsymbol{p}\cdot\frac{\partial \psi}{\partial \boldsymbol{x}} + \boldsymbol{f}\cdot\frac{\partial \psi}{\partial \boldsymbol{p}} = \frac{\delta \psi}{\delta t}$$

Problem 3

The Ergodic Problem (Is N Big Enough)



1. A family has two children.

2. One child is a girl.

What are the odds that the family has two girls?

a) 1/4

b) 1/3

c) 1/2

1. One in a hundred people typically test positive for COVID in Casablanca.

2. Captain Renault uses a self-test that is 95% accurate.

3. He tests positive.

What are the odds that Captain Renault has the virus?

a) 95/100

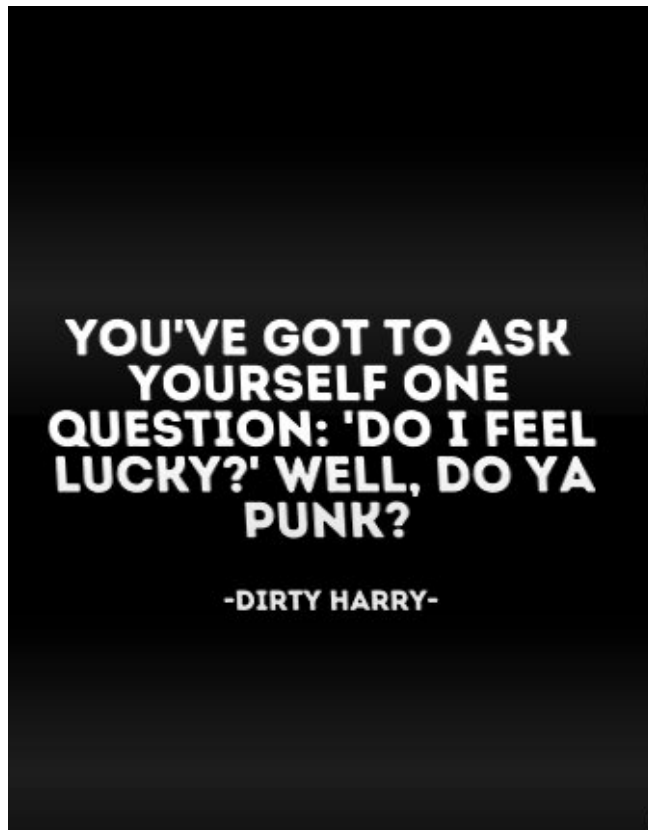
b) 16/100

c) 1/100

Problem 3

Solved?

The Ergodic Problem (Is N Big Enough?)



Renault: It might be a good idea for you to disappear from Montréal for awhile. There's a Free French garrison over in Brazzaville. I could be induced to arrange a passage.

Rick: My letter of transit? You still owe me 10,000 francs!

Renault: And that 10,000 francs should just pay our expenses.

Rick : Our expenses?