PHY 3700 meets PHY 6756

Lecture 2: Spherically-Symmetric Applications

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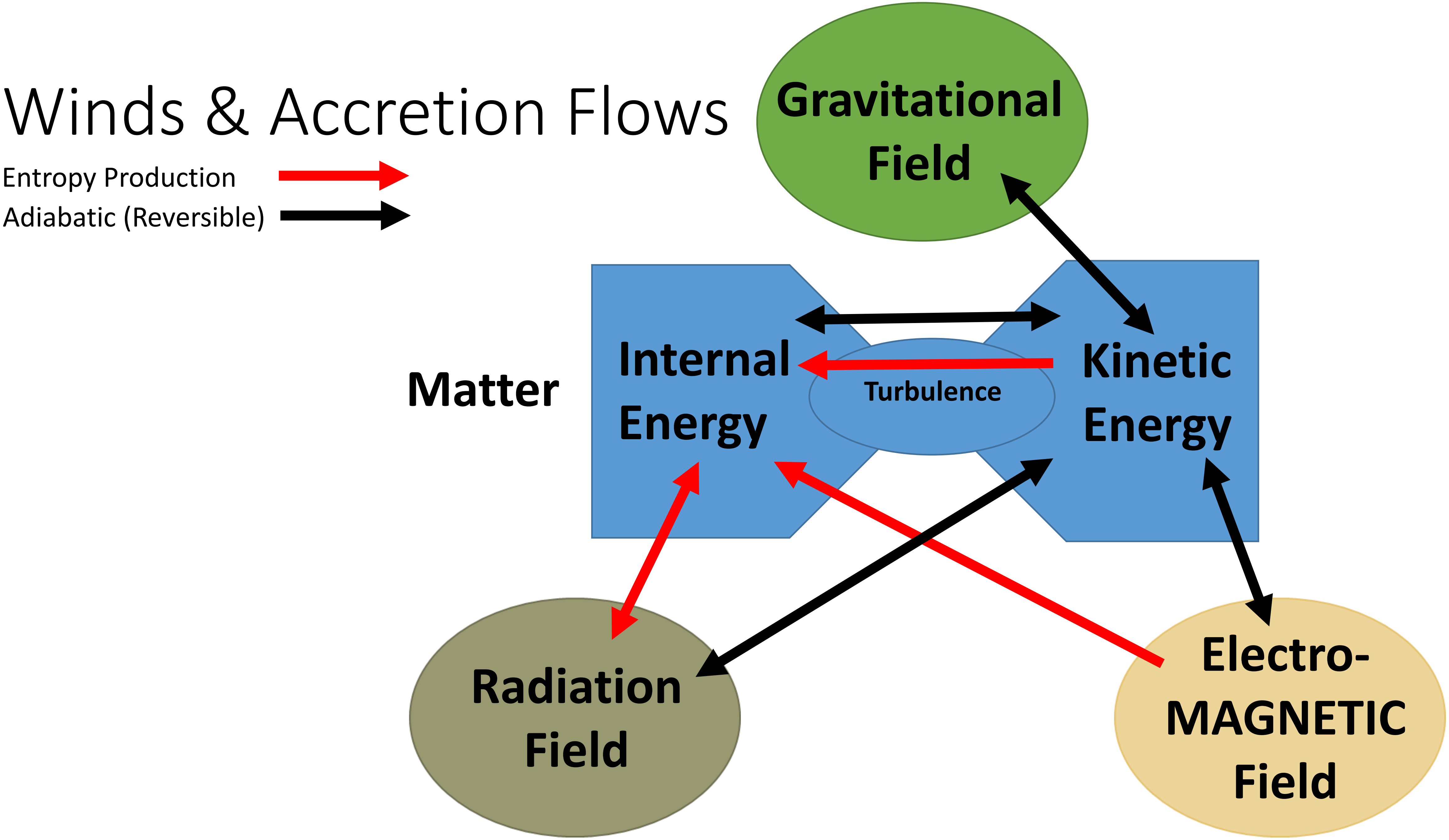
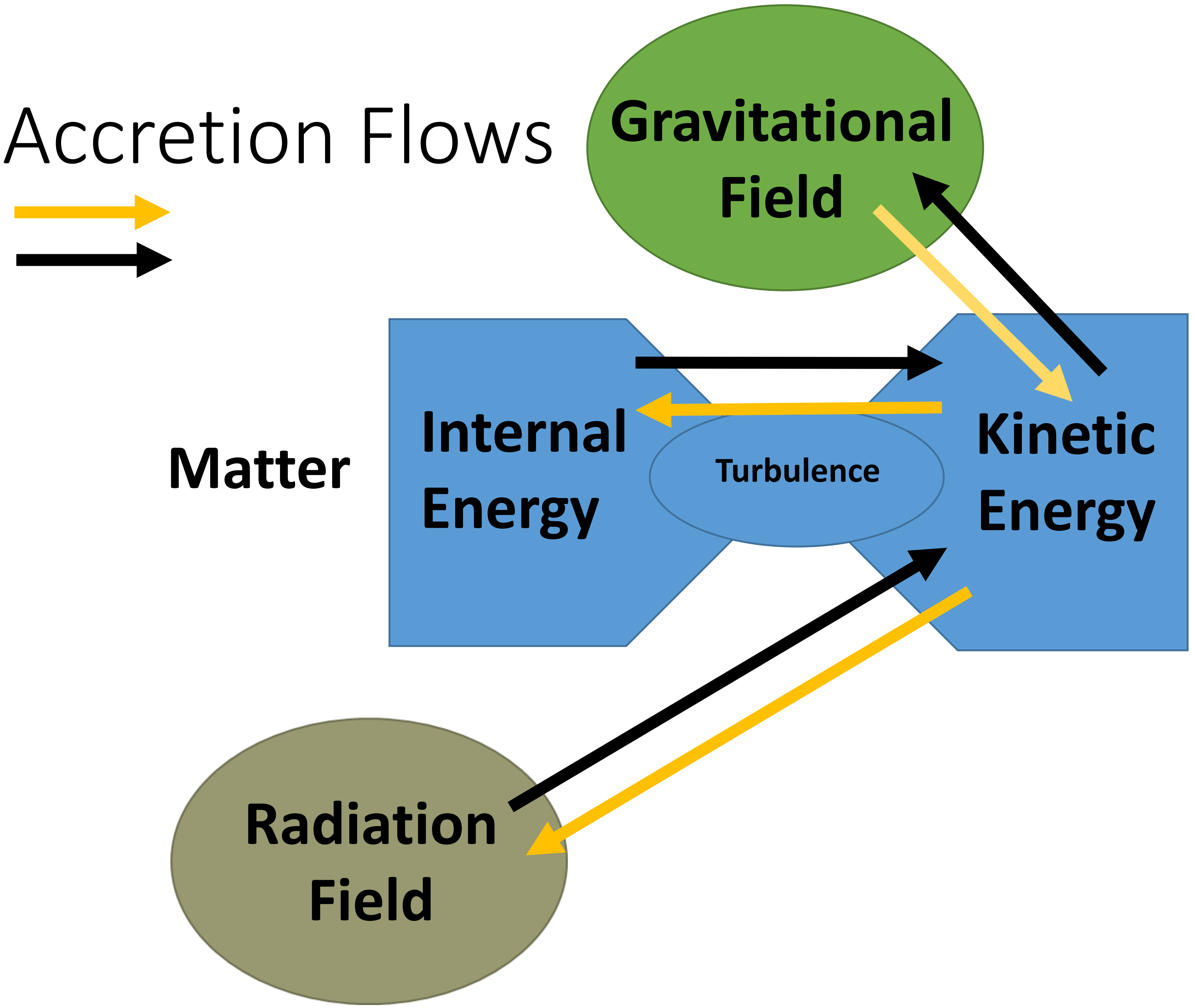
Winds & Accretion Flows



Winds & Accretion Flows

Accretion Flows

Winds



It is very hard to do one-dimensional MHD in spherical geometry. Sorry.

Momentum Bookkeeping

$$\frac{\partial}{\partial t}\rho\boldsymbol{u} + \cdot\ \boldsymbol{\nabla}\cdot(p + \rho\boldsymbol{u}\boldsymbol{u}) = -\rho\ \boldsymbol{\nabla}\Phi + \boldsymbol{f}$$

$$\boldsymbol{\nabla}\cdot\mathbb{G} = \rho\boldsymbol{\nabla}\Phi$$

$$\frac{1}{c^2}\cdot\frac{\partial \boldsymbol{F}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbb{P} = \frac{1}{c}\int_0^\infty d\nu \int d\mathbf{n}\ \mathbf{n}[\eta_\nu - \chi_\nu I_\nu]$$

$$\frac{\partial \boldsymbol{\mathfrak{P}}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbb{M} = 0$$

Energy Bookkeeping

$$\frac{\partial}{\partial t}\frac{1}{2}\rho\|\boldsymbol{u}\|^2 + \boldsymbol{\nabla}\cdot\frac{1}{2}\rho\|\boldsymbol{u}\|^2\,\boldsymbol{u} = -\boldsymbol{u}\cdot\boldsymbol{\nabla}p - \rho\boldsymbol{u}\cdot\boldsymbol{\nabla}\Phi + \boldsymbol{u}\cdot\boldsymbol{f}$$

$$\frac{\partial}{\partial t}\rho e + \boldsymbol{\nabla}\cdot(\rho e + p)\boldsymbol{u} = +\boldsymbol{u}\cdot\boldsymbol{\nabla}p + \rho T\dot{s}$$

$$\frac{\partial}{\partial t}\frac{1}{2}\rho\Phi + \boldsymbol{\nabla}\cdot(\rho\Phi\boldsymbol{u} + \boldsymbol{G}) = \rho\boldsymbol{u}\cdot\boldsymbol{\nabla}\Phi$$

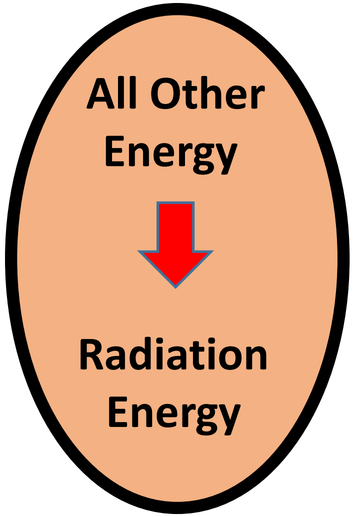
$$\frac{\partial E}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{F} = \int_0^\infty d\nu\int d\mathbf{n}\,[\eta_\nu - \chi_\nu I_\nu]$$

$$\frac{\partial\mathfrak{E}}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{\mathfrak{F}} = 0$$

The Radiation Field (Review)

$$\frac{1}{c}\cdot\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I_\nu = \eta_\nu - \chi_\nu I_\nu$$

Gray Atmosphere Approximation



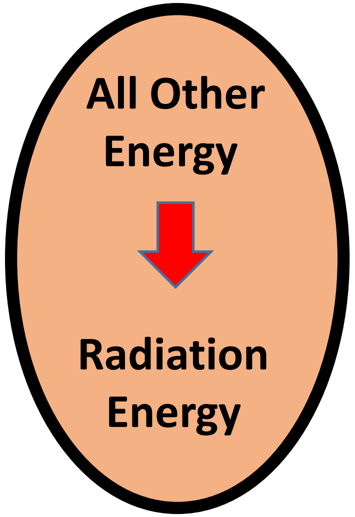
Coupling to matter

$$\frac{\partial E}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{F} = \kappa\left[4\sigma_R T^4 - cE\right] + \kappa\frac{1}{c}\boldsymbol{u}\cdot\boldsymbol{F} + \cdots$$

$$\frac{1}{c^2}\cdot\frac{\partial \boldsymbol{F}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbb{P} = -\frac{\kappa}{c}\left[\boldsymbol{F} - \boldsymbol{u}\left\{\frac{4\sigma_R}{c}T^4 + \mathbb{P}\right\}\right] + \cdots$$

Someone needs to tell us how to determine the radiation pressure tensor!

The Radiation Field---Radiation Diffusion



$$\frac{1}{c}\cdot\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I_\nu = \eta_\nu - \chi_\nu I_\nu$$

Gray Atmosphere Approximation

$$4\sigma_R T^4 \approx cE'$$
$$\nabla\cdot\mathbb{P}' \approx -\frac{\kappa}{c}\boldsymbol{F}'$$
$$\mathbb{P}' \approx \frac{1}{3}E'\,\mathbb{I}$$

$u/\ell c$ versus $1/\lambda$

$$\frac{\partial E}{\partial t} + \nabla\cdot\boldsymbol{F} = \kappa\,[4\sigma_R T^4 - cE] + \kappa\frac{1}{c}\boldsymbol{u}\cdot\boldsymbol{F} + \cdots$$

$$\frac{1}{c^2}\cdot\frac{\partial \boldsymbol{F}}{\partial t} + \nabla\cdot\mathbb{P} = -\frac{\kappa}{c}\left[\boldsymbol{F} - \boldsymbol{u}\left\{\frac{4\sigma_R}{c}T^4 + \mathbb{P}\right\}\right] + \cdots$$

Radiation ***pressure***, “thermal” ***conduction*** and ***viscosity*** emerge in the laboratory frame!

The Radiation Field (Review)

$$\frac{1}{c}\cdot\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\nabla I_\nu = \eta_\nu - \chi_\nu I_\nu$$

Pure Thomson Scattering

All Other Energy

Radiation Energy

Coupling to matter

$$\frac{\partial E}{\partial t} + \nabla\cdot\boldsymbol{F} = \cdots$$

$$\frac{1}{c^2}\cdot\frac{\partial \boldsymbol{F}}{\partial t} + \nabla\cdot\mathbb{P} = -\frac{\sigma}{c}\left[\boldsymbol{F} - \boldsymbol{u}\{\tfrac{2}{3}E + \mathbb{P}\}\right] + \cdots$$

Someone needs to tell us how to determine the radiation pressure tensor!

Radiative Force and Entropy Generation

$$\boldsymbol{f} = \frac{\kappa}{c}\left[\boldsymbol{F} - \boldsymbol{u}\left\{\frac{4\sigma_R}{c}T^4 + \mathbb{P}\right\}\right] + \frac{\sigma}{c}\left[\boldsymbol{F} - \boldsymbol{u}\left\{\frac{2}{3}E + \mathbb{P}\right\}\right]$$

Emission/Absorption

Scattering

$$\rho T\dot{s} = -\kappa\left[4\sigma_R T^4 - cE\right] - (2\kappa + \sigma)\frac{1}{c}\boldsymbol{u}\cdot\boldsymbol{F}$$

Emission/Absorption

Scattering

$$\frac{\partial}{\partial t}\boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\frac{1}{\rho}\nabla p - \nabla\Phi + \frac{1}{\rho}\boldsymbol{f}$$

$$\frac{\partial}{\partial t}e + \boldsymbol{u}\cdot\nabla e + \cdot\frac{p}{\rho}\nabla\cdot\boldsymbol{u} = T\dot{s}$$

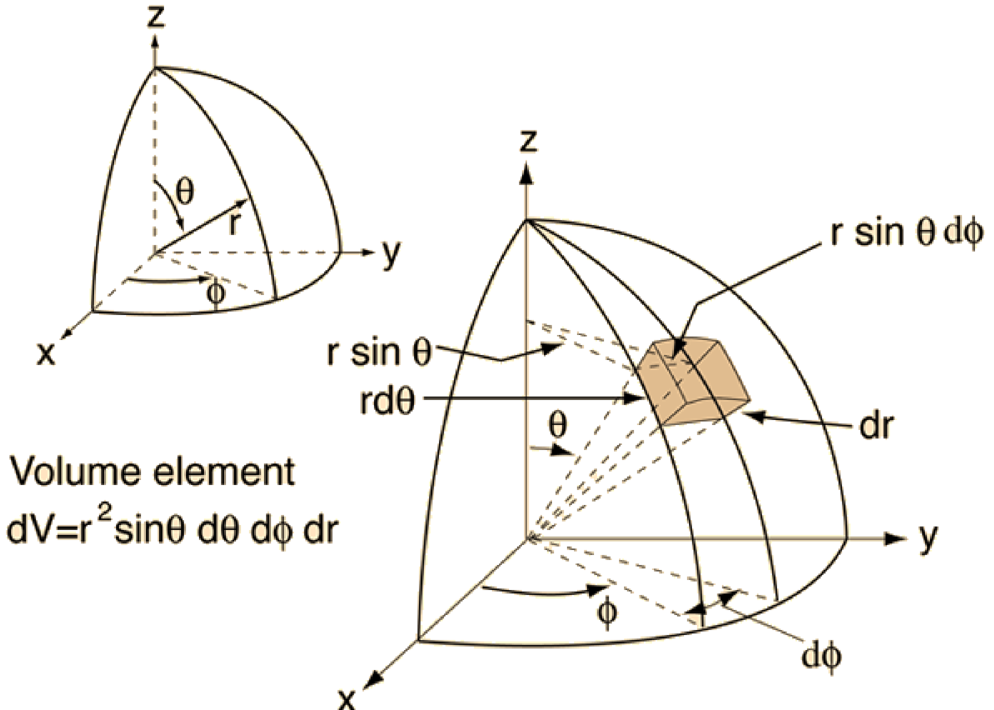
$$\frac{p}{\rho} = (\gamma - 1)c_V T = (\gamma - 1)e = \frac{\mathcal{R}}{\mu}T$$

Steady Spherical Flows

$$\nabla \cdot \rho \boldsymbol{u} = 0 \qquad \nabla \cdot \mathfrak{F} = 0$$

$$\nabla \cdot \{\frac{1}{2}\rho||\boldsymbol{u}||^2\, \boldsymbol{u} + (\rho e + p)\boldsymbol{u} + [\rho\Phi\boldsymbol{u} + \boldsymbol{G}] + \boldsymbol{F}\,\} = 0$$

Kinetic Thermal Gravitational Radiation



$$\dot{M} = 4\pi r^2 \rho u$$

$$\dot{E} = \dot{M}\{\frac{1}{2}u^2 + e + \frac{p}{\rho} + \Phi\} + 4\pi r^2 F$$

Nice---we have two constants from our conserved fluxes of energy and mass!!

One Equation Short...

$$p = (\gamma - 1)\rho c_V T = (\gamma - 1)\rho e = \frac{\mathcal{R}}{\mu}\rho T$$

$$\dot{M} = 4\pi r^2 \rho u$$

$$\dot{E} = \dot{M}\{\frac{1}{2}u^2 + e + \frac{p}{\rho} + \Phi\} + 4\pi r^2 F$$

$$\mathcal{L}_*$$

$$\Phi = -\frac{GM_*}{r}$$

We need one more relation between ρ, u, and e!

$$\boldsymbol{u} \cdot \boldsymbol{\nabla u} = -\frac{1}{\rho}\boldsymbol{\nabla}p - \boldsymbol{\nabla}\Phi + \frac{1}{\rho}\boldsymbol{f}$$

Nice---we have two constants from our conserved fluxes of energy and mass!!

The Bondi/Parker Equation-I

$$\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} = -\frac{1}{\rho}\boldsymbol{\nabla}p - \boldsymbol{\nabla}\Phi + \frac{1}{\rho}\boldsymbol{f}$$

$$u\frac{du}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{d\Phi}{dr} + \frac{1}{\rho}\cdot f$$

$$u\frac{du}{dr} = -\frac{d}{dr}\frac{p}{\rho} - \frac{p}{\rho^2}\frac{d\rho}{dr} - \frac{GM_*}{r^2} + \frac{1}{\rho}\cdot f$$

$$u\frac{du}{dr} = \boxed{-\frac{da^2}{dr} + \frac{2a^2}{r} + \frac{a^2}{u}\frac{du}{dr}} - \frac{GM_*}{r^2} + \frac{1}{\rho}\cdot f$$

Radiative Terms Enter Here!

$$\frac{p}{\rho} = (\gamma-1)c_V T = (\gamma-1)e = \frac{\mathcal{R}}{\mu}T \equiv a^2$$

$$\boldsymbol{u}\cdot\boldsymbol{\nabla}e + \cdot\frac{p}{\rho}\boldsymbol{\nabla}\cdot\boldsymbol{u} = T\dot{s}$$

Someone needs to tell us how to determine the isothermal sound speed!

The Internal Energy Equation

$$\boldsymbol{u} \cdot \boldsymbol{\nabla} e + \cdot \frac{p}{\rho} \boldsymbol{\nabla} \cdot \boldsymbol{u} = T\dot{s}$$

$$u \frac{de}{dr} - \frac{p}{\rho^2} \frac{d\rho}{dr} = T\dot{s}$$

$$\frac{1}{\gamma - 1} u \frac{da^2}{dr} + u \frac{2a^2}{r} + a^2 \frac{du}{dr} = T\dot{s}$$

Radiative Terms Enter Here!

$$-\frac{da^2}{dr} = (\gamma - 1)[\frac{2a^2}{r} + \frac{a^2}{u} \frac{du}{dr}] - \frac{1}{u}(\gamma - 1)T\dot{s}$$

This indeed tells us how to determine the isothermal sound speed!

The Bondi/Parker Equation(s)-II

Kinetic Thermal Gravitational Radiation

$$\dot{E} = \dot{M}\{\frac{1}{2}u^2 + \frac{\gamma}{\gamma - 1}a^2 - \frac{GM}{r}_*\} + 4\pi r^2 F$$

$$\dot{M} = 4\pi r^2 \rho u$$

I. $$u\frac{du}{dr} = -\frac{da^2}{dr} + [\frac{2a^2}{r} + \frac{a^2}{u}\frac{du}{dr}] - \frac{GM_*}{r^2} + \frac{1}{\rho}\cdot f$$

II. $$-\frac{da^2}{dr} = (\gamma - 1)[\frac{2a^2}{r} + \frac{a^2}{u}\frac{du}{dr}] - \frac{1}{u}(\gamma - 1)T\dot{s}$$

Radiative Terms Enter Here!

Now all we need are the radiative terms!

Woops, what happens if $\gamma=1$?!

The Bondi/Parker Equation-No Radiation

Now evaluate the energy and the mass flux at the critical point where $r=r_c$ and $u=u_c$. If you can also determine $a(r_c)$ and $\rho(r_c)$ by some means---***voilà***!!

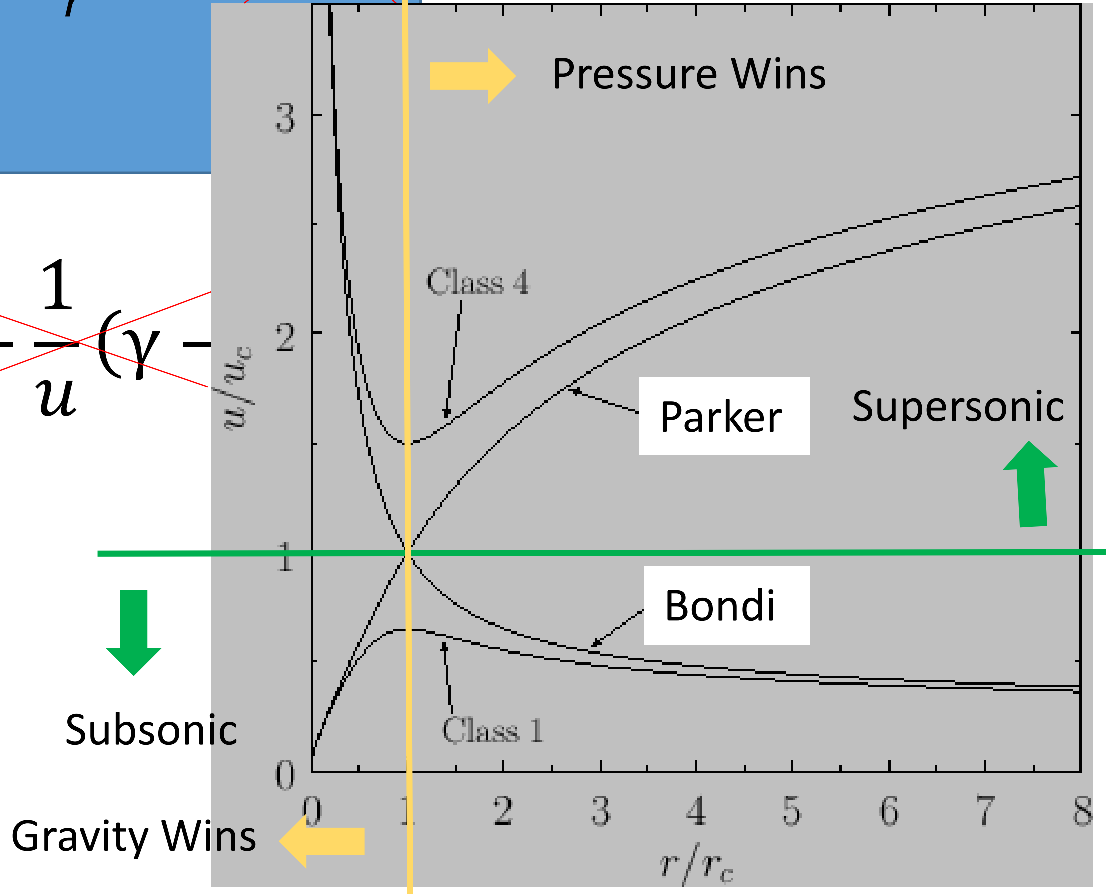
Kinetic Thermal Gravitational ~~Radiation~~

$$\dot{E} = \dot{M}\left\{\frac{1}{2}u^2 + \frac{\gamma}{\gamma-1}a^2 - \frac{GM}{r}\right\} + 4\pi r^2 F$$

$$\dot{M} = 4\pi r^2 \rho u$$

I. $$u\frac{du}{dr} - \gamma\frac{a^2}{u}\frac{du}{dr} = \gamma\frac{2a^2}{r} - \frac{GM}{r^2} + \frac{1}{\rho}\cdot f - \frac{1}{u}(\gamma - \ldots$$ [illegible]

$$\left(u - \gamma\frac{a^2}{u}\right)\frac{du}{dr}$$



Isothermal Wind & Accretion

$$\dot{E} = \dot{M}\{\frac{1}{2}u^2 + \frac{\gamma}{\gamma-1}a^2 - \frac{GM_*}{r}\} + 4\pi r^2 F$$

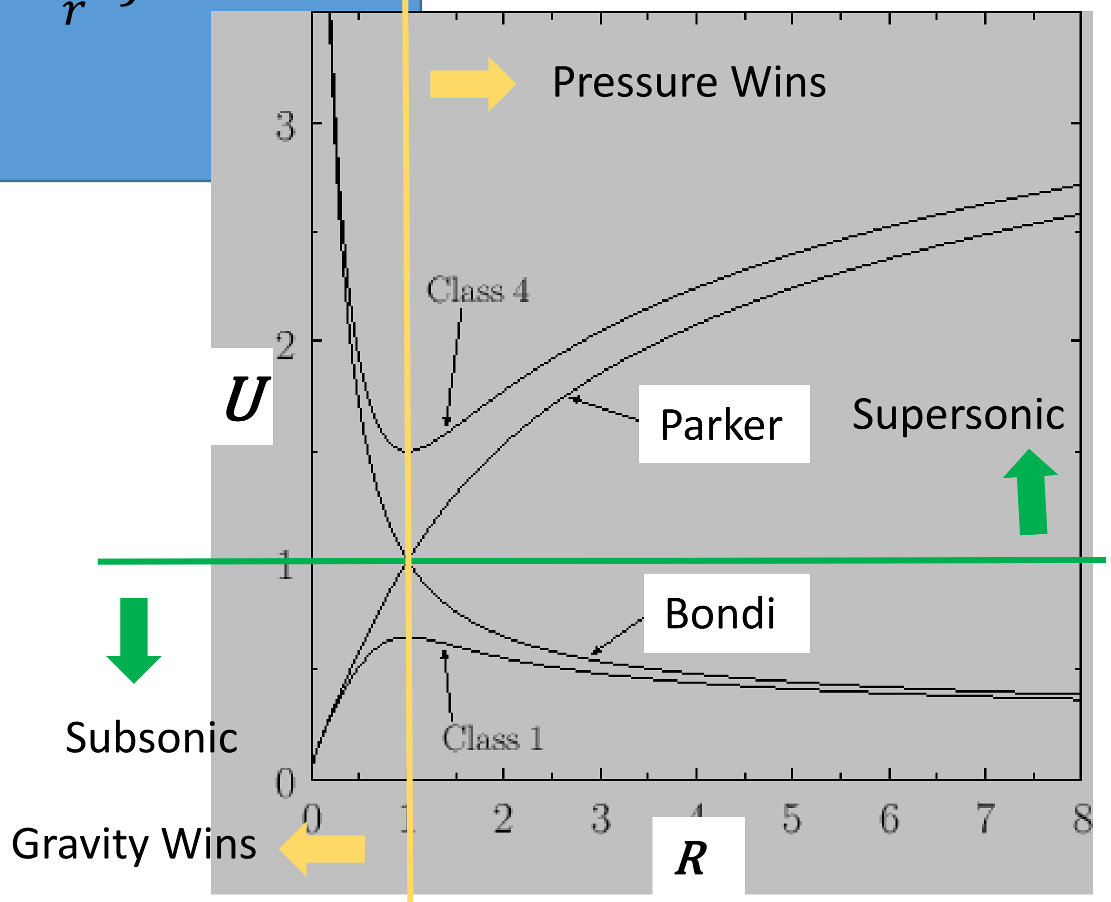
$$\dot{M} = 4\pi r^2 \rho u$$

Now just some constant!

$$u\frac{du}{dr} - \frac{a^2}{u}\frac{du}{dr} = \frac{2a^2}{r} - \frac{GM_*}{r^2}$$

$$\frac{d}{dr}[\frac{u^2}{2} - a^2 \log u] = \frac{d}{dr}[2a^2 \log r + \frac{GM_*}{r}]$$

$$\frac{1}{U}\exp\frac{1}{2}U^2 = R^2 \exp[\frac{2}{R} - \frac{3}{2}]$$



The Bondi/Parker Equation(s)-III

Kinetic Thermal Gravitational Radiation

$$\dot{E} = \dot{M}\{\frac{1}{2}u^2 + \frac{\gamma}{\gamma-1}a^2 - \frac{GM_*}{r}\} + 4\pi r^2 F$$

$$\dot{M} = 4\pi r^2 \rho u$$

I. $$u\frac{du}{dr} = -\frac{da^2}{dr} + [\frac{2a^2}{r} + \frac{a^2}{u}\frac{du}{dr}] - \frac{GM_*}{r^2} + \frac{1}{\rho}\cdot f$$

II. $$-\frac{da^2}{dr} = (\gamma-1)[\frac{2a^2}{r} + \frac{a^2}{u}\frac{du}{dr}] - \frac{1}{u}(\gamma-1)T\dot{s}$$

Radiative Terms Enter Here!

Radiatively Driven Winds-Phenomenology

Kinetic Thermal Gravitational Radiation

$$\dot{E} = \dot{M}\{\frac{1}{2}u^2 + \frac{\gamma}{\gamma-1}a^2 - \frac{GM_*}{r}\} + 4\pi r^2 F$$

$$\dot{M} = 4\pi r^2 \rho u$$

I. $$u\frac{du}{dr} - \gamma\frac{a^2}{u}\frac{du}{dr} = \gamma\frac{2a^2}{r} - \frac{GM_*}{r^2} + \frac{1}{\rho}\cdot f - \frac{1}{u}(\gamma - 1)T\dot{s}$$

$$u\frac{du}{dr} - \gamma\frac{a^2}{u}\frac{du}{dr} = \gamma\frac{2a^2}{r} - \frac{GM_*}{r^2} + \frac{C}{r^2} + Lu\frac{du}{dr}$$

$$\left[u\frac{du}{dr}\right]^{\alpha}$$

CAK Theory of Stellar Winds

Continuum Scattering

Resonance Line Absorption

Radiatively Driven Winds-Phenomenology

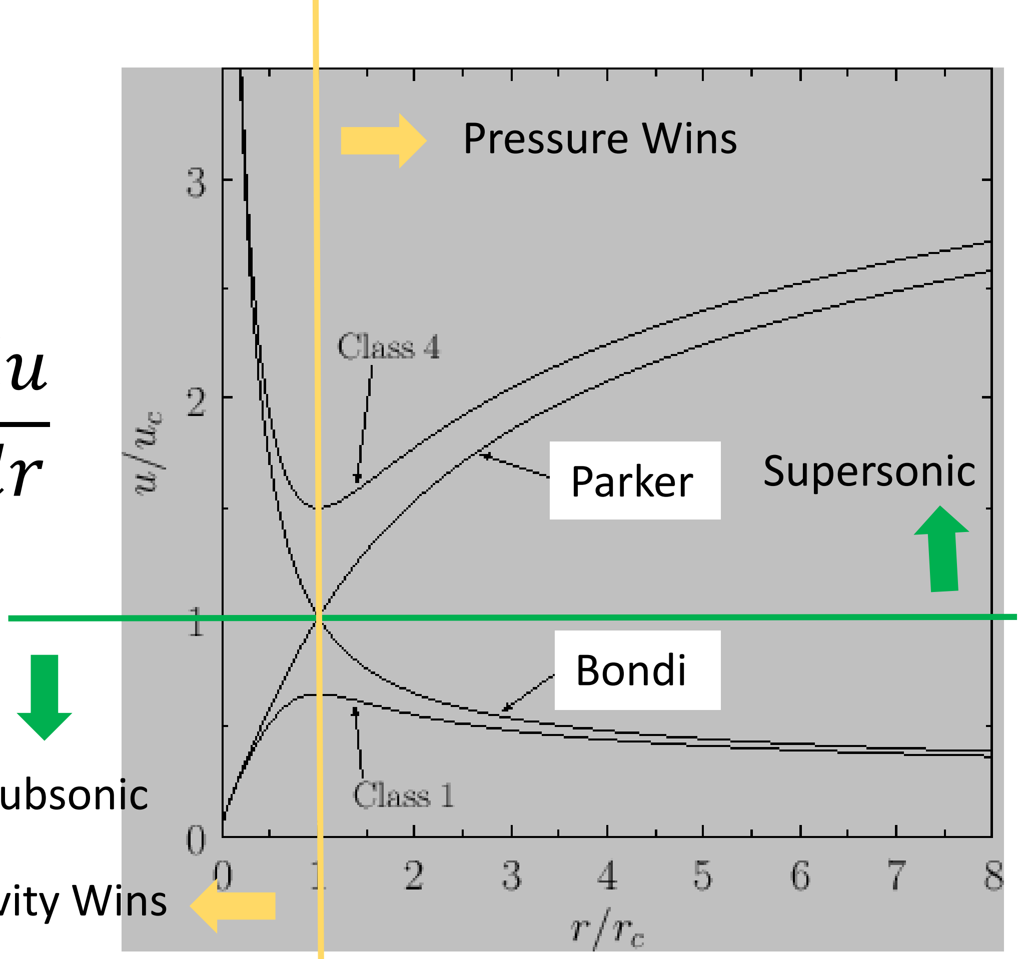
As this approaches unity we reach the "Eddington Limit" for accretion.

$$\text{I.}\quad u\frac{du}{dr} - \gamma\frac{a^2}{u}\frac{du}{dr} = \gamma\frac{2a^2}{r} - \frac{GM_*}{r^2} + \frac{C}{r^2} + Lu\frac{du}{dr}$$

$$\left(u(1-L) - \frac{\gamma a^2}{u}\right)\frac{du}{dr} = \gamma\frac{2a^2}{r} - (1-\varepsilon)\frac{GM_*}{r^2}$$

Line Absorption ***increases*** the critical speed.

Continuum Scattering ***reduces*** the critical radius.



Flows with Thomson Scattering

...but what about the energy density in the radiation field?

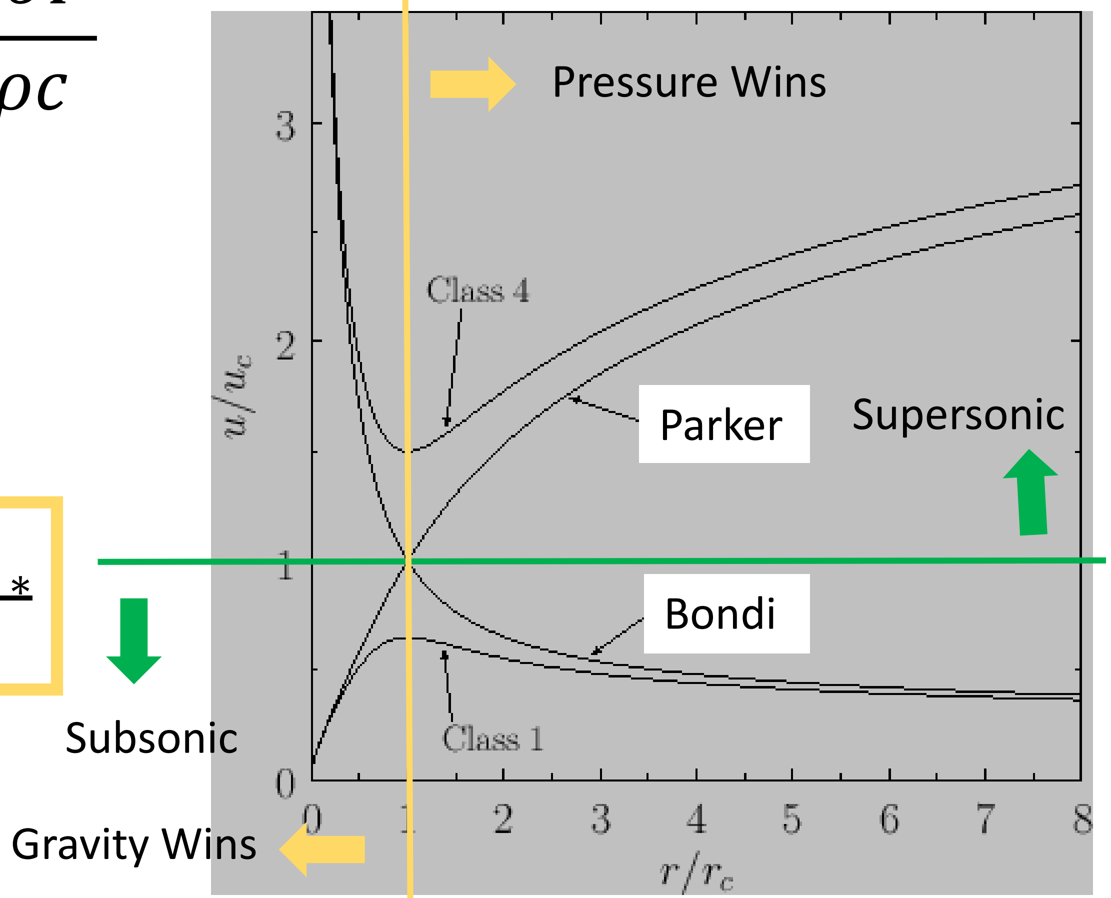
$$\sigma = \frac{\sigma_T \rho}{m} \qquad F = \frac{\mathcal{L}_*}{4\pi r^2}$$

$$\varepsilon = \frac{\mathcal{L}_*}{\mathcal{L}_E} \qquad \frac{C}{r^2} = \frac{\gamma \sigma F}{\rho c}$$

I. $$u\frac{du}{dr} - \gamma\frac{a^2}{u}\frac{du}{dr} = \gamma\frac{2a^2}{r} - \frac{GM_*}{r^2} + \frac{C}{r^2}$$

$$\left(u - \frac{\gamma a^2}{u}\right)\frac{du}{dr} = \gamma\frac{2a^2}{r} - (1-\varepsilon)\frac{GM_*}{r^2}$$

Continuum Scattering ***reduces*** the critical radius.



Thomson Scattering-The Rest of the Story

$$\nabla \cdot \mathbb{P} = -\frac{\sigma}{c}\boldsymbol{F} + \cdots$$

$$\frac{\mathrm{d}P_{rr}}{\mathrm{d}r} + \frac{3P_{rr} - E}{r} = -\frac{\sigma F}{c} \quad \begin{cases} \sigma = \dfrac{\sigma_T \rho}{m} \\ F = \dfrac{\mathcal{L}_*}{4\pi r^2} \end{cases}$$

The Eddington Approximation

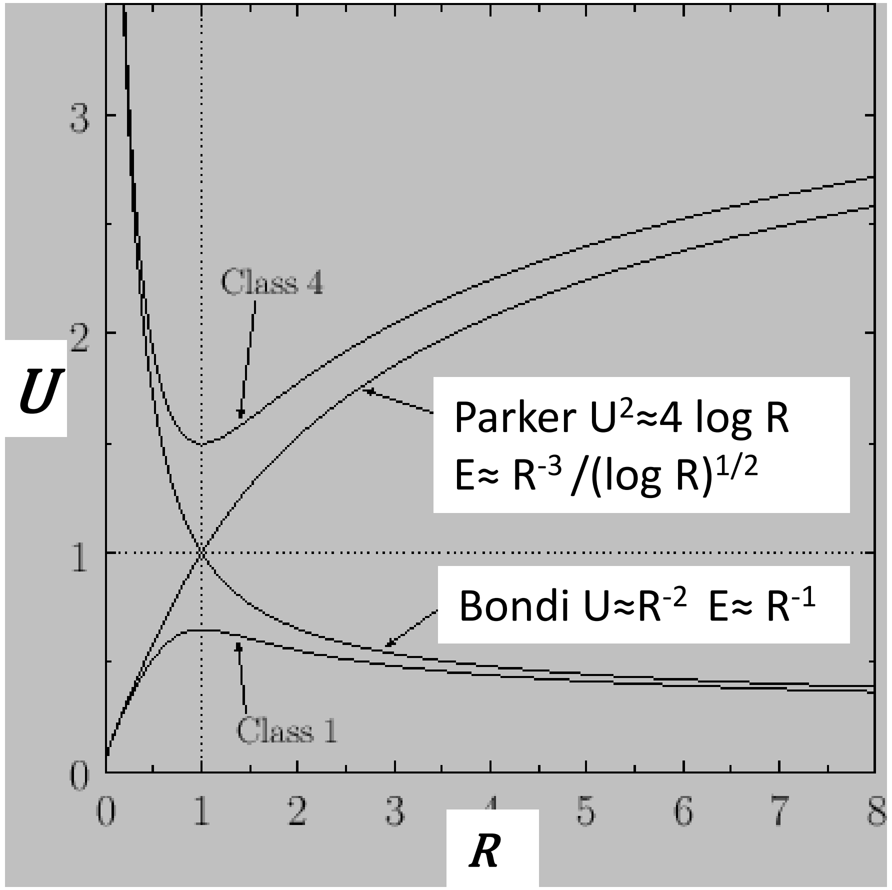
$$E = P_{rr} + P_{\theta\theta} + P_{\phi\phi} = 3P_{rr}$$

$$\frac{\mathrm{d}E}{\mathrm{d}r} = -\frac{3\rho\sigma_T\mathcal{L}_*}{4\pi r^2 mc}$$

$$\frac{\mathrm{d}E}{\mathrm{d}r} = -\frac{3\dot{M}\sigma_T\mathcal{L}_*}{16\pi^2 r^4 umc}$$

For the Bondi Accretion Flow $F/E = cH/J$ tends to ***zero*** as r goes to ***infinity.*** For the Parker Wind $F/E = cH/J$ tends to ***infinity*** as r goes to ***infinity…***<u>but</u> cH/J cannot exceed c !!!! **Now what???**

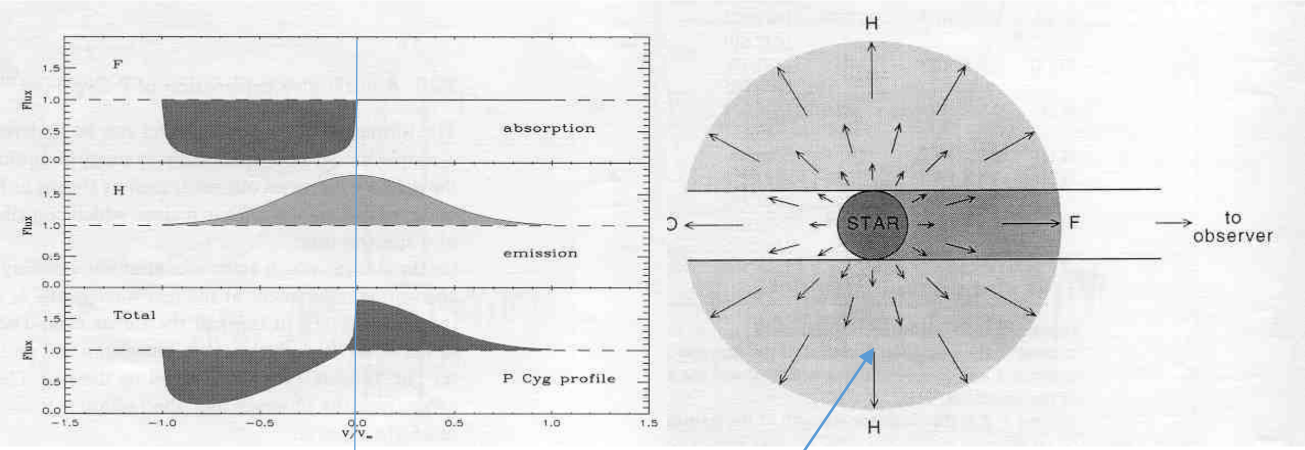
$$\frac{1}{U}\exp\frac{1}{2}U^2 = R^2\exp\left[\frac{2}{R} - \frac{3}{2}\right]$$



Radiatively Driven Winds-CAK Theory

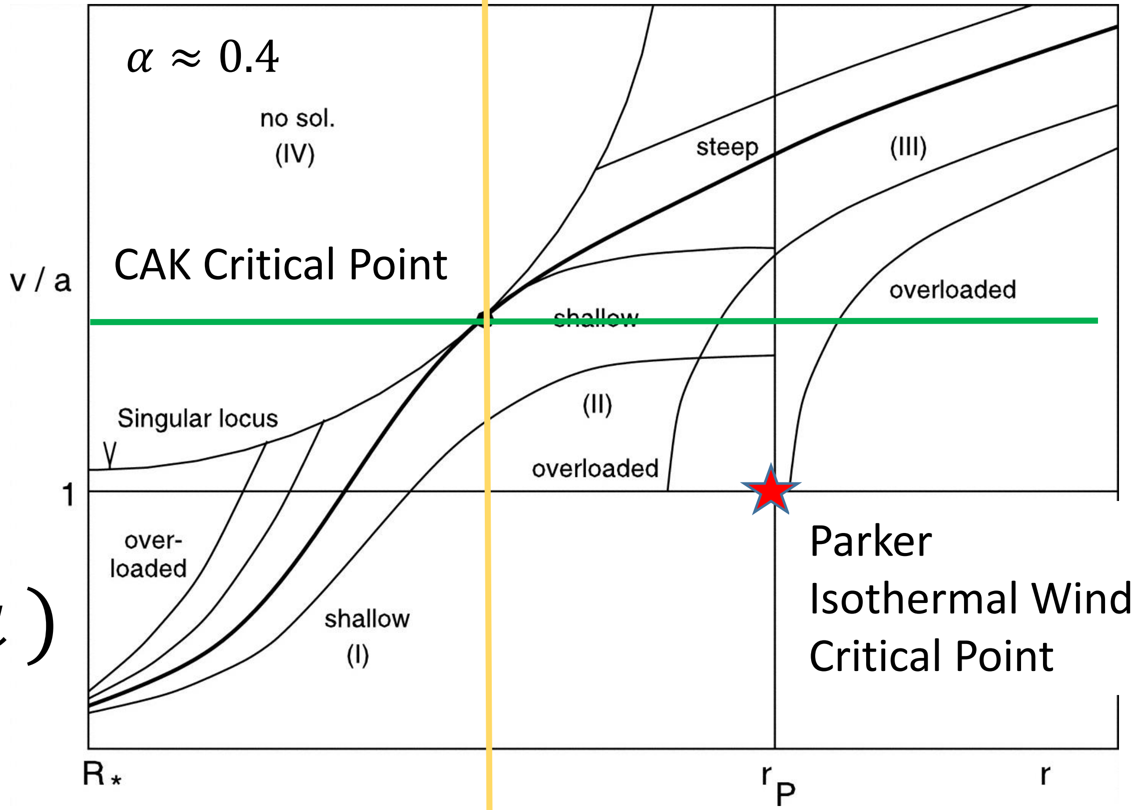
P Cygni

$$u\frac{du}{dr} - \gamma\frac{a^2}{u}\frac{du}{dr} - L\left[u\frac{du}{dr}\right]^{\alpha} = \gamma\frac{2a^2}{r} - \frac{GM_*}{r^2}$$

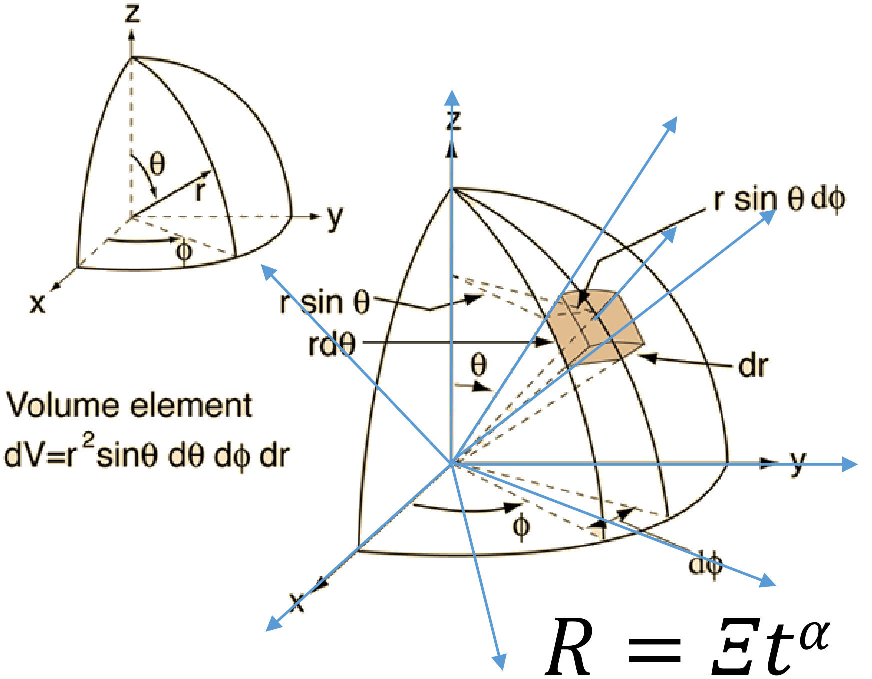


Line Center

$$\nu' = \Gamma_u\, \nu\, \left(1 - \frac{1}{c}\,\mathbf{n}\cdot \boldsymbol{u}\right)$$



Blast Waves-I



$$R = \Xi t^{\alpha}$$

$$(r,t) \to (\xi,t) \qquad \xi = \frac{r}{t^{\alpha}} \qquad \Xi = \frac{R}{t^{\alpha}}$$

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + \frac{\partial \xi}{\partial t}\frac{\partial}{\partial \xi} \qquad \frac{\partial}{\partial t} + \frac{\xi}{t}\frac{\partial}{\partial \xi}$$

$$\frac{\partial}{\partial r} \to \frac{\partial \xi}{\partial r}\frac{\partial}{\partial \xi} \qquad \frac{1}{t^{\alpha}}\frac{\partial}{\partial \xi}$$

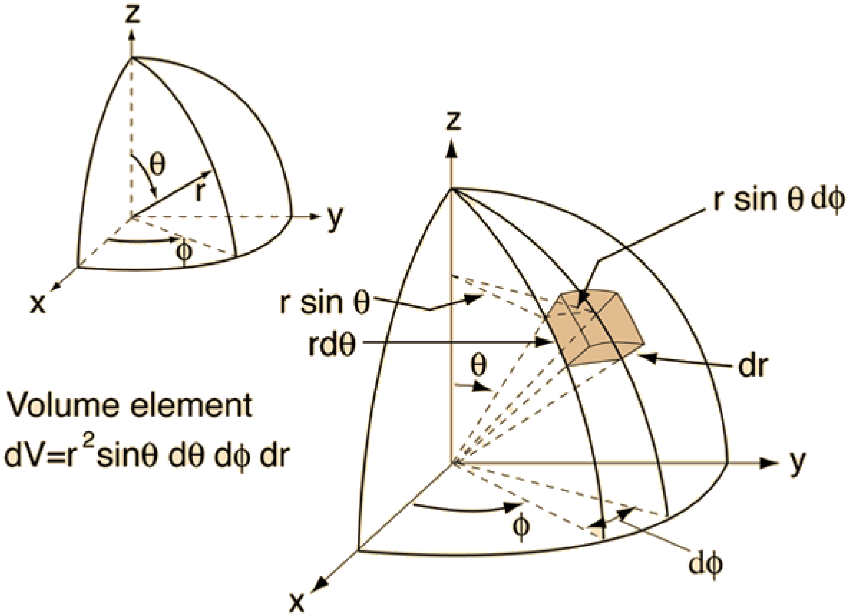
$$\frac{\partial}{\partial t} + u\frac{\partial}{\partial r} \to \frac{\partial}{\partial t} + \frac{\xi}{t}\frac{\partial}{\partial \xi} + \frac{u}{t^{\alpha}}\frac{\partial}{\partial \xi} \Rightarrow u \to t^{\alpha-1}U(\xi) \quad \textbf{I.}$$

Kinetic $$4\pi \int_0^{\Xi t^{\alpha}} dr\, r^2\, \frac{1}{2}\rho u^2 = 4\pi t^{5\alpha\text{-}2} \int_0^{\Xi} d\xi\, \xi^2\, \frac{1}{2}\rho U^2$$

Thermal $$4\pi \int_0^{\Xi t^{\alpha}} dr\, r^2\, \rho e = 4\pi t^{3\alpha} \int_0^{\Xi} d\xi\, \xi^2\, \rho e \Rightarrow e \to t^{2\alpha-2}E(\xi) \quad \textbf{II.}$$

Gravitational $$4\pi \int_0^{\Xi t^{\alpha}} dr\, r^2\, \rho\Phi = -4\pi G M t^{2\alpha} \int_0^{\Xi} d\xi\, \xi\, \rho$$

Blast Waves-II



$$(r,t) \to (\xi,t) \qquad \xi = \frac{r}{t^\alpha} \qquad \Xi = \frac{R}{t^\alpha}$$

$$R = \Xi t^\alpha$$

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + \frac{\partial \xi}{\partial t}\frac{\partial}{\partial \xi} \qquad \frac{\partial}{\partial t} + \frac{\xi}{t}\frac{\partial}{\partial \xi}$$

$$\frac{\partial}{\partial r} \to \frac{\partial \xi}{\partial r}\frac{\partial}{\partial \xi} \qquad \frac{1}{t^\alpha}\frac{\partial}{\partial \xi}$$

$$\frac{\partial}{\partial t} + u\frac{\partial}{\partial r} \to \frac{\partial}{\partial t} + \frac{\xi}{t}\frac{\partial}{\partial \xi} + \frac{u}{t^\alpha}\frac{\partial}{\partial \xi} \Rightarrow u \to t^{\alpha-1}U(\xi) \quad \textbf{I.}$$

III. $\frac{p}{\rho} = (\gamma - 1)e$

$$\rho \to t^\sigma D(\xi) \Rightarrow p \to t^{2\alpha+\sigma-2}P(\xi)$$

Kinetic $$4\pi \int_0^{\Xi t^\alpha} dr\, r^2\, \frac{1}{2}\rho u^2 = 4\pi t^{5\alpha+\sigma\text{-}2} \int_0^{\Xi} d\xi\, \xi^2\, \frac{1}{2} D U^2$$

Thermal $$4\pi \int_0^{\Xi t^\alpha} dr\, r^2\, \rho e = 4\pi t^{5\alpha+\sigma\text{-}2} \int_0^{\Xi} d\xi\, \xi^2\, DE \Rightarrow e \to t^{2\alpha-2}E(\xi) \quad \textbf{II.}$$

Gravitational $$4\pi \int_0^{\Xi t^\alpha} dr\, r^2\, \rho\Phi = -4\pi G M t^{2\alpha+\sigma} \int_0^{\Xi} d\xi\, \xi\, D$$

Blast Waves-III

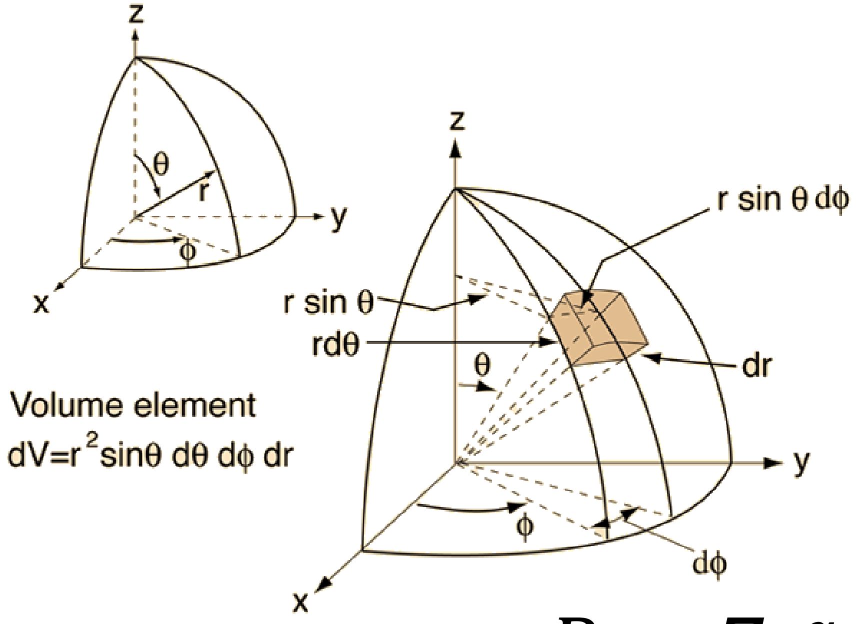
$$(r,t) \rightarrow (\xi,t) \qquad \xi = \frac{r}{t^{\alpha}} \qquad \Xi = \frac{R}{t^{\alpha}}$$

$$u \rightarrow t^{\alpha-1}U(\xi)$$

$$e \rightarrow t^{2\alpha-2}E(\xi)$$

$$\rho \rightarrow t^{\sigma}D(\xi)$$

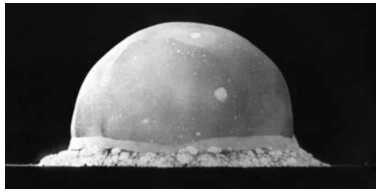
$$p \rightarrow t^{2\alpha+\sigma-2}P(\xi)$$



$$R = \Xi t^{\alpha}$$

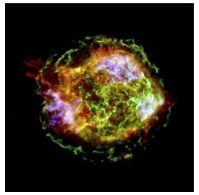
With these energy conservation scalings in place, we end up with a coupled set of ***ODE***s for *U, E, D* and *P.*

$$\alpha = \frac{2}{5} \qquad \sigma = 0$$



The Sedov/Taylor/von Neumann Self-Similar Blast Wave Solution for a ***non-gravitating***, ***uniform***,***cold*** medium.

$$\alpha = \frac{2}{3} \qquad \sigma = -\frac{4}{3}$$



The Sedov/Taylor/von Neumann Self-Similar Blast Wave modified to include gravity, and a $1/r^2$ background density falloff.

Equation of State $$P = D(\gamma - 1)E$$

Kinetic $$4\pi \int_0^{\Xi t^{\alpha}} dr\, r^2\, \frac{1}{2}\rho u^2 = 4\pi t^{5\alpha+\sigma\text{-}2} \int_0^{\Xi} d\xi\, \xi^2\, \frac{1}{2} D U^2$$

Thermal $$4\pi \int_0^{\Xi t^{\alpha}} dr\, r^2\, \rho e = 4\pi t^{5\alpha+\sigma\text{-}2} \int_0^{\Xi} d\xi\, \xi^2\, DE$$

Gravitational $$4\pi \int_0^{\Xi t^{\alpha}} dr\, r^2\, \rho\Phi = -4\pi G M t^{2\alpha+\sigma} \int_0^{\Xi} d\xi\, \xi\, D$$

Sedov/Taylor/von Neumann

$$(r,t) \rightarrow (\xi,t)$$

$$\xi = \frac{r}{t^{\alpha}}$$

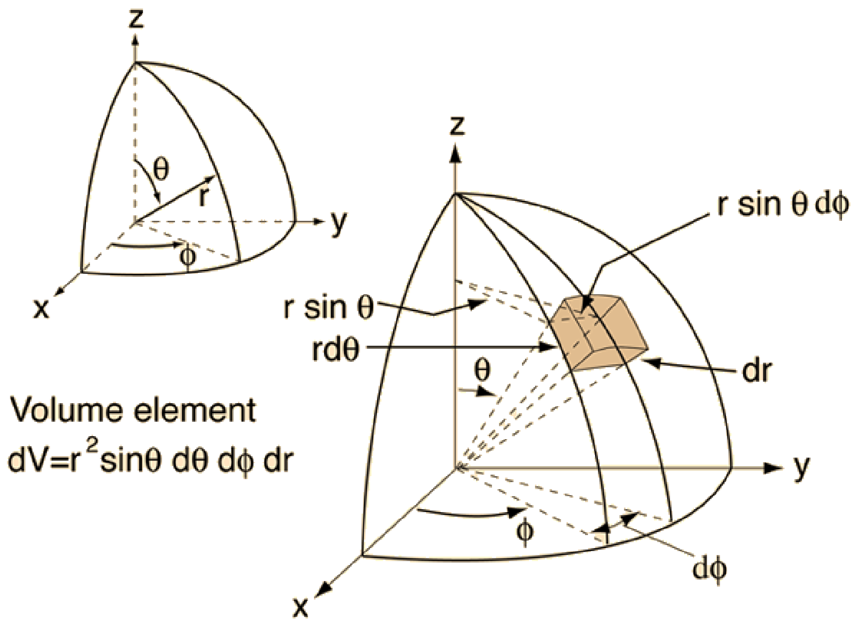
$$\Xi = \frac{R}{t^{\alpha}}$$

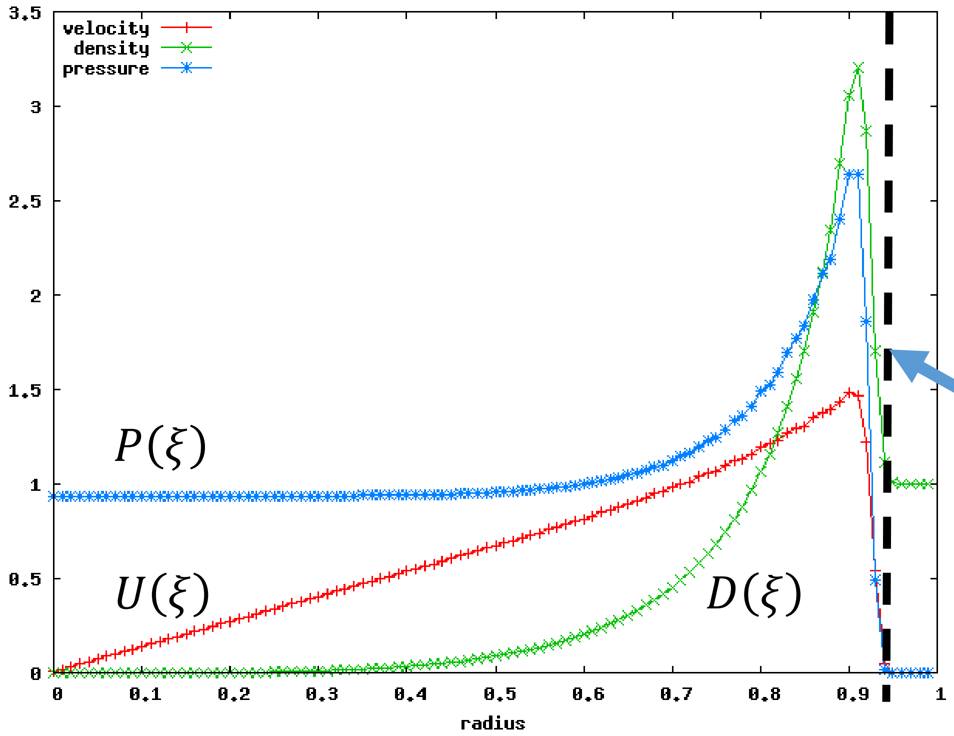
$$u \rightarrow t^{-2/5}U(\xi)$$

$$e \rightarrow t^{-4/5}E(\xi)$$

$$\rho \rightarrow D(\xi)$$

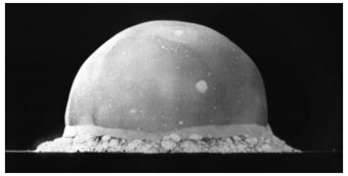
$$p \rightarrow t^{-4/5}P(\xi)$$





$$R = \Xi t^{2/5}$$

$$\alpha = \frac{2}{5} \qquad \sigma = 0$$



The Sedov/Taylor/von Neumann Self-Similar Blast Wave Solution for a ***non-gravitating***, ***uniform***, ***cold*** medium.

This is a shock front propagating into a gas of ***constant*** density and ***zero*** temperature, so not matter how slowly it moves, it propagates with an ***infinite*** upstream Mach number, and a fixed, finite, downstream Mach number that depends on γ.

What's Next?

- Try out Bondi and Parker including the Newton/Poisson Equation
- Try an optically-thick gray atmosphere radiation driven wind
- Try out a different equation of state that includes ionization/recombination
- Generalize to 2D Axisymmetric Winds and Accretion Flows with MHD
- Look for a Self-Similar Blast Wave with the Radiation Field included
- See what happens when u/c becomes of order unity
- What types of waves live in RMHD
- …

Now, you are only astrophysically limited by your own imagination!

Merci! Au Revoir.