PHY 3700 meets PHY 6756

Lecture 1: Radiation Magnetohydrodynamics

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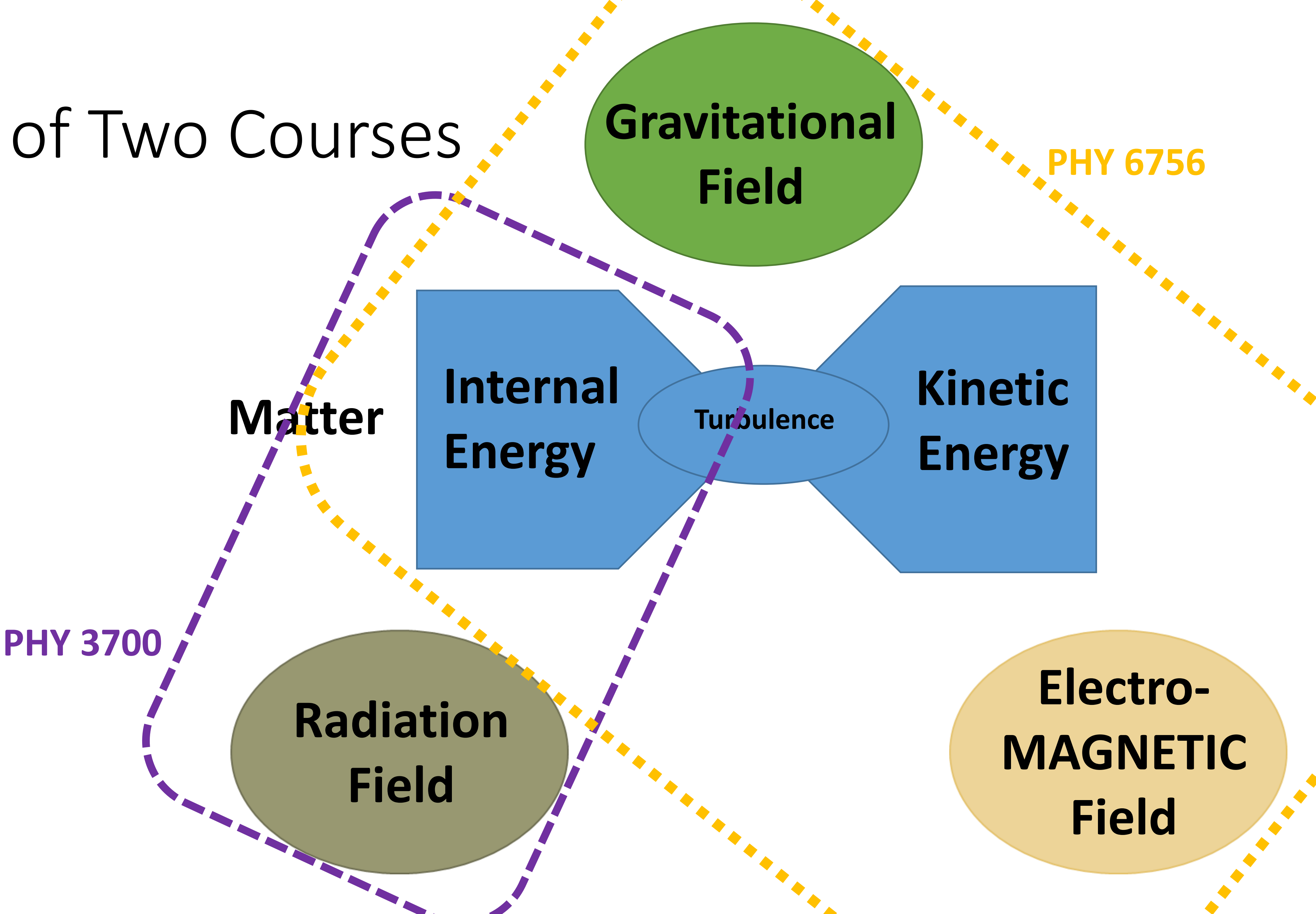
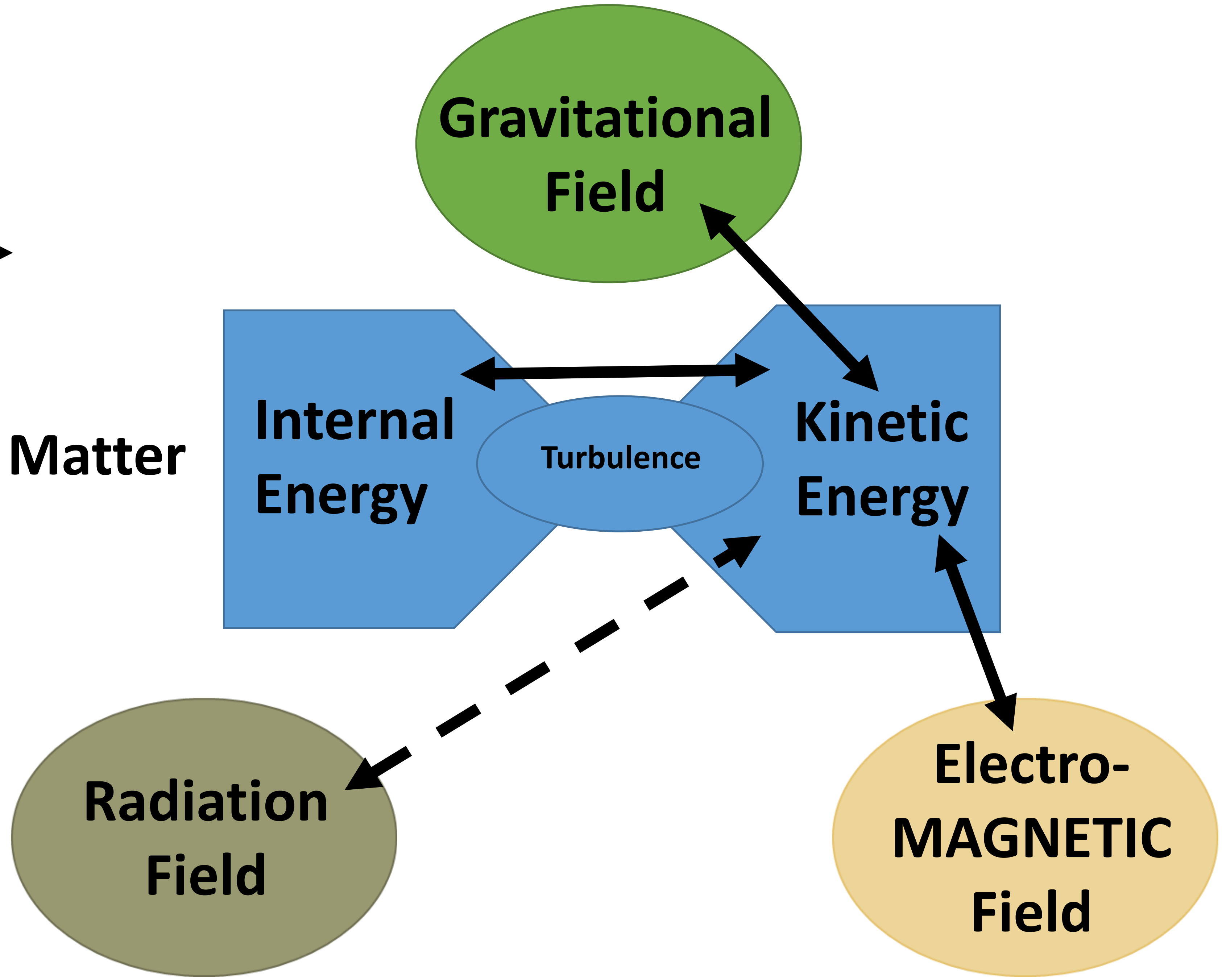
A Tale of Two Courses



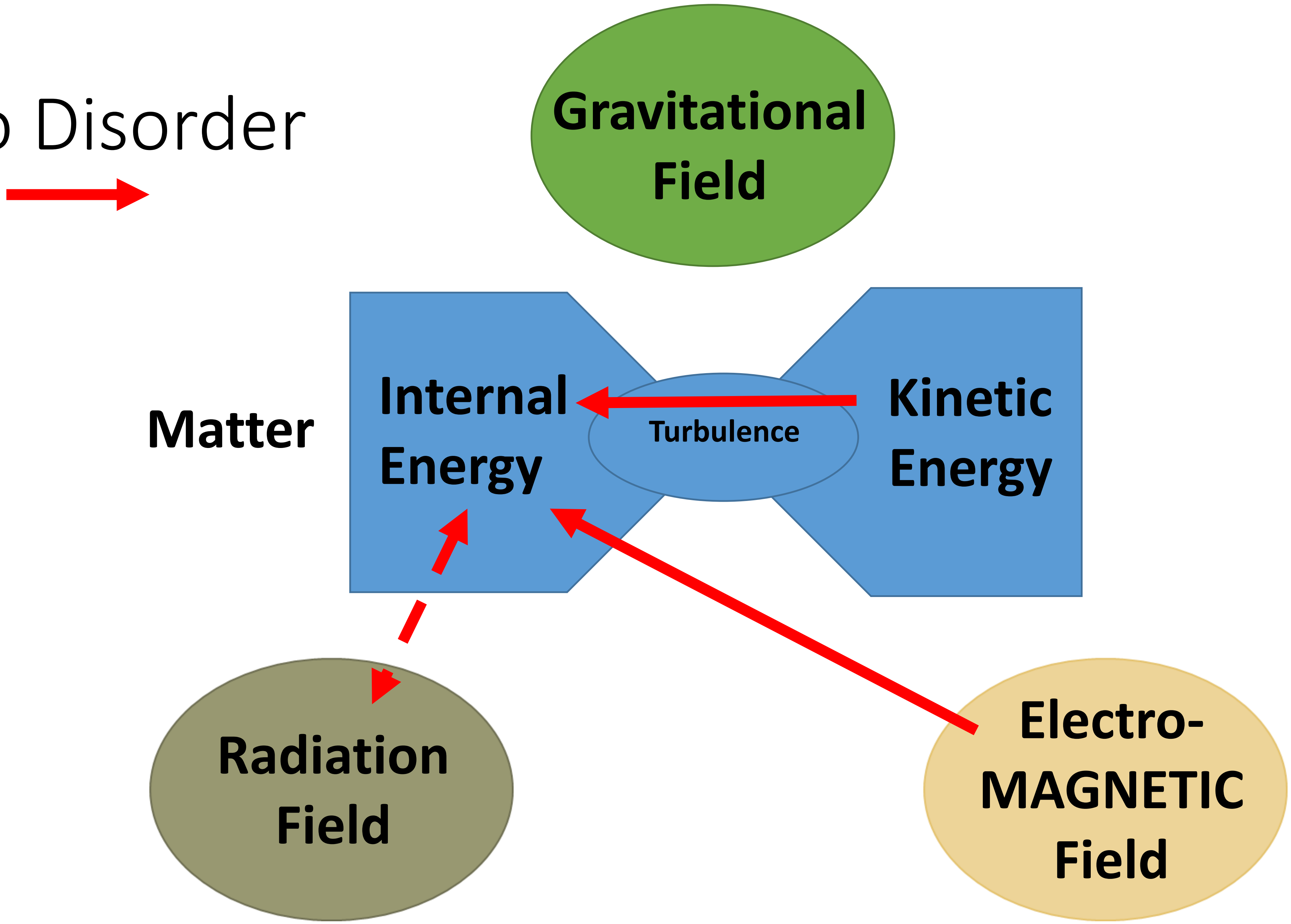
Attribution

Adiabatic (Reversible) →



Adding to Disorder

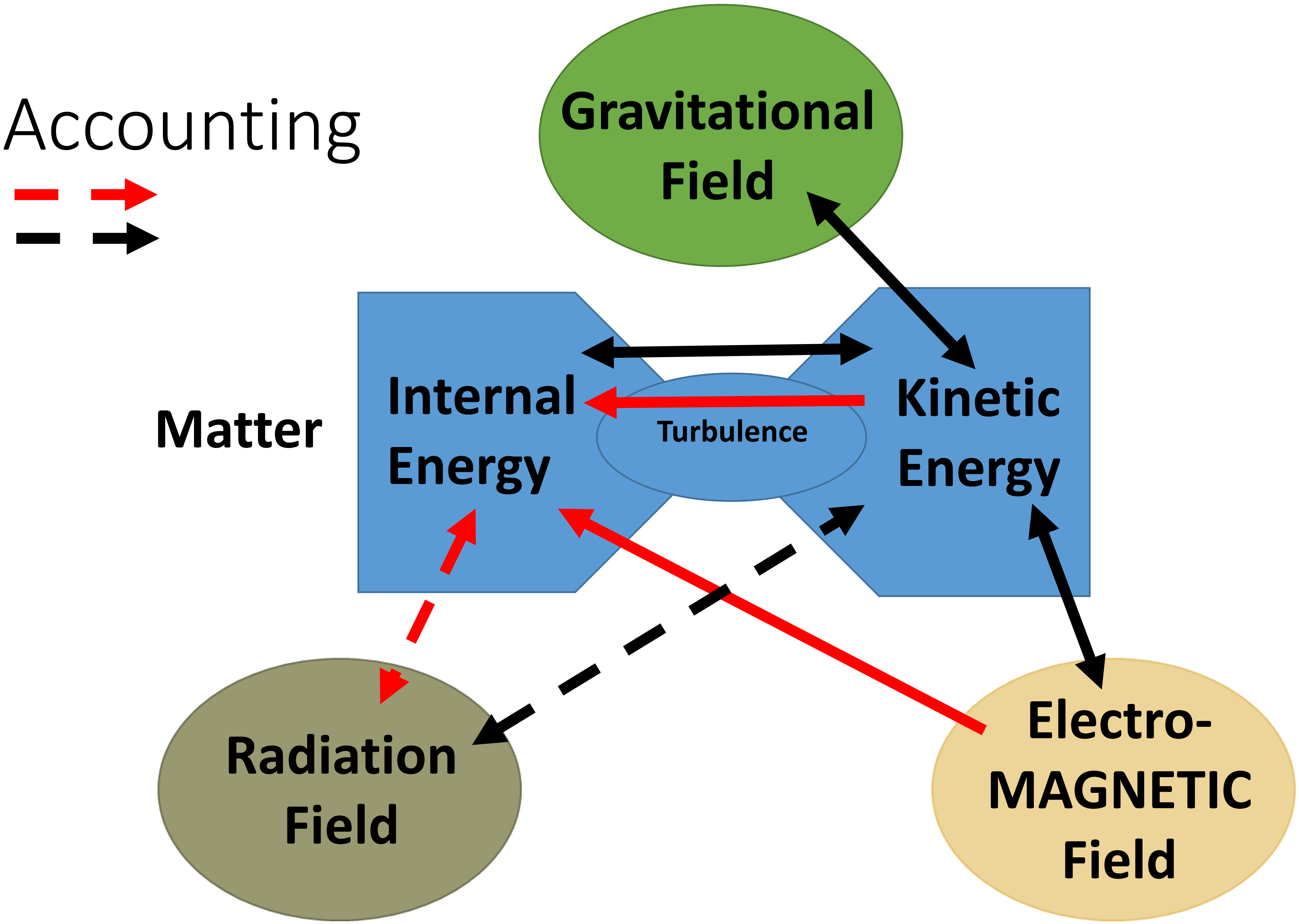
Entropy Production



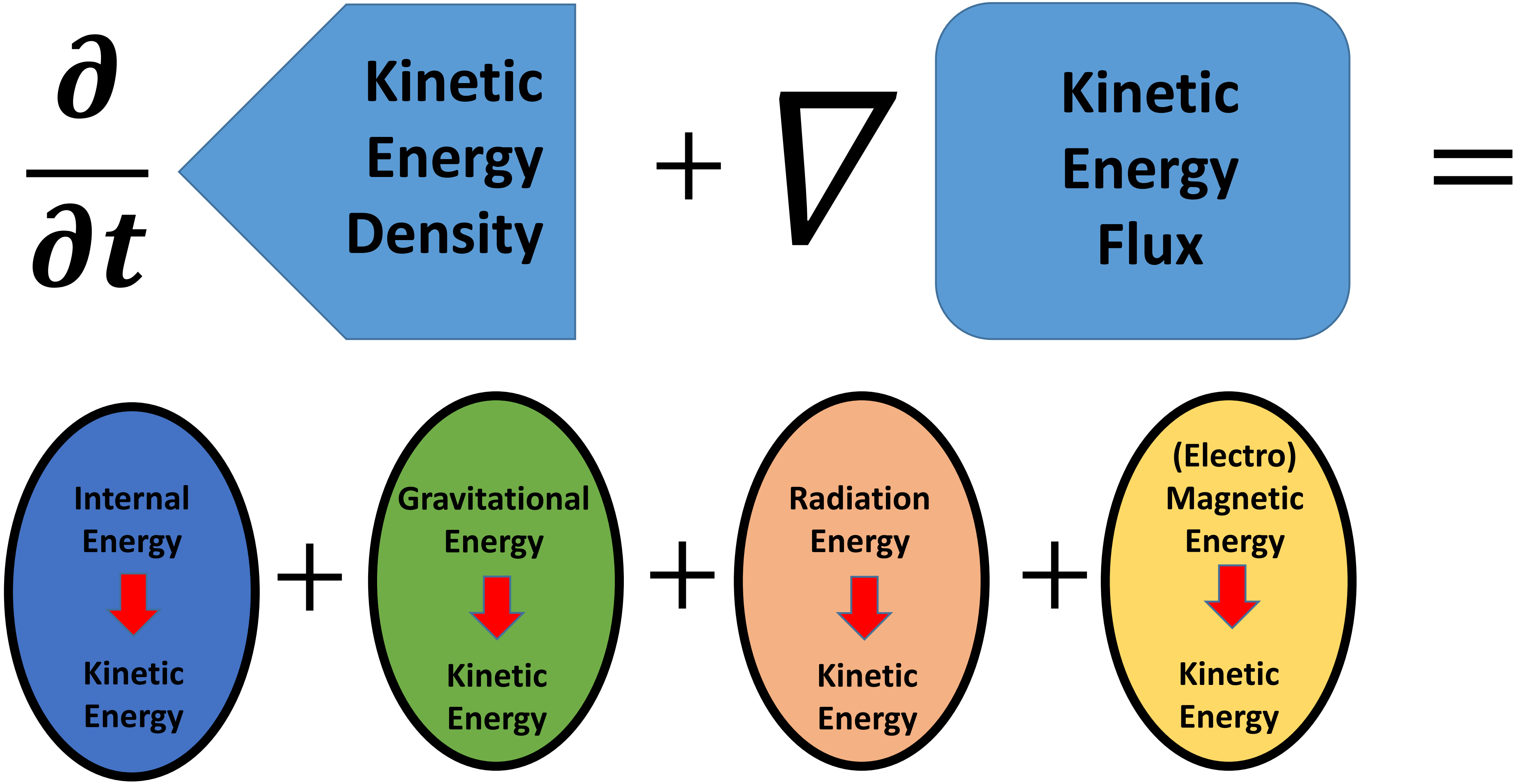
Full Cost Accounting

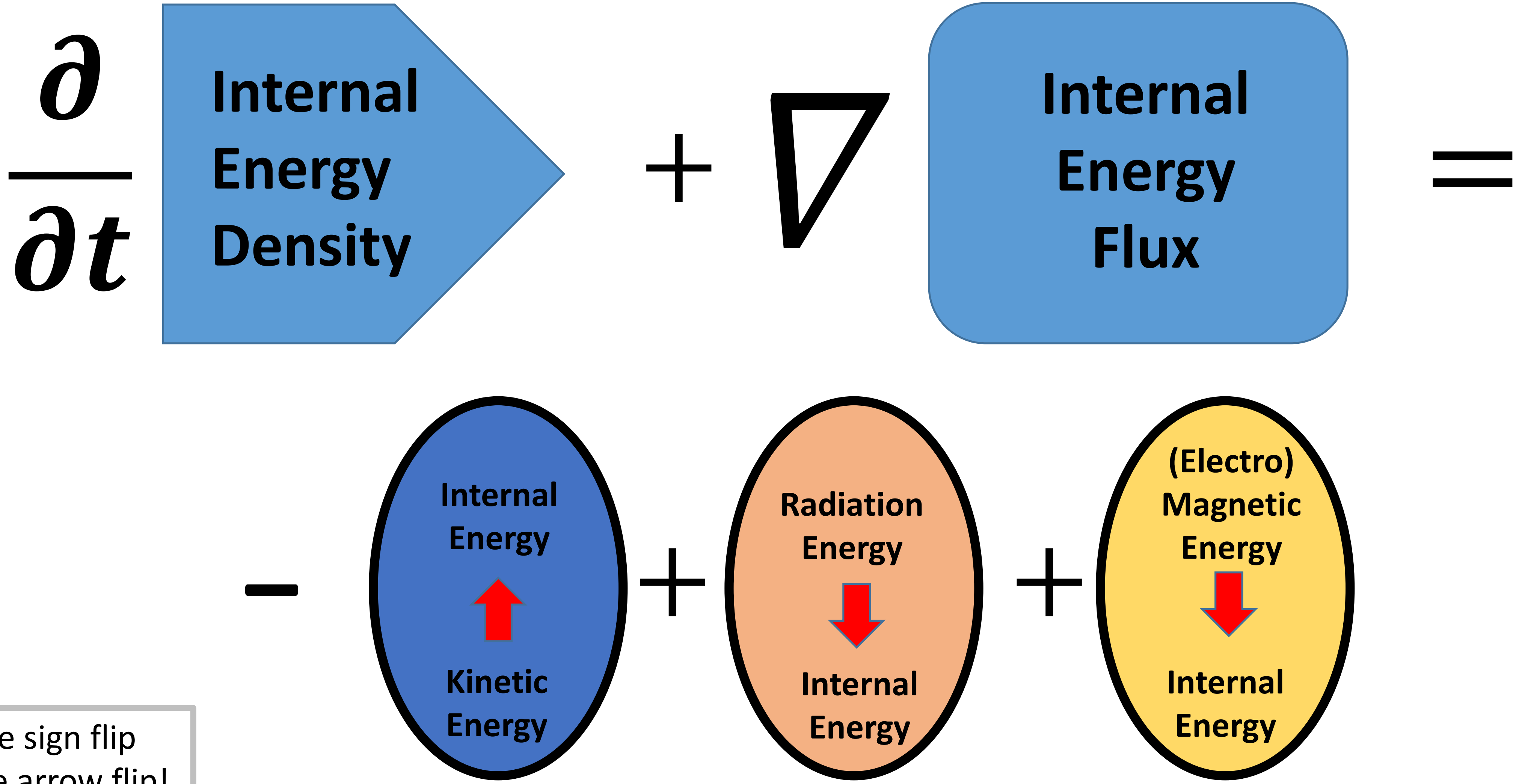
Entropy Production

Adiabatic (Reversible)



Symbolic Conservation Laws



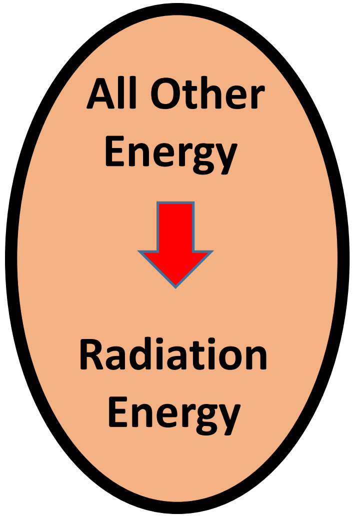


Note the sign flip
with the arrow flip!

The Radiation Field

Coupling to matter

$$\frac{1}{c}\cdot\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\boldsymbol{\nabla} I_\nu = \eta_\nu - \chi_\nu I_\nu$$



scalings: *u/ℓc* *1/ℓ* *?* *1/λ*

$$\frac{\partial E}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{F} = \int_0^\infty d\nu \int d\mathbf{n}\,[\eta_\nu - \chi_\nu I_\nu]$$

$$\frac{1}{c^2}\cdot\frac{\partial \boldsymbol{F}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbb{P} = \frac{1}{c}\int_0^\infty d\nu \int d\mathbf{n}\,\mathbf{n}[\eta_\nu - \chi_\nu I_\nu]$$

Someone needs to tell us how to determine the radiation pressure tensor!

The Electromagnetic Field (Review)

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho_e \qquad c\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad c\nabla \times \boldsymbol{B} = 4\pi \boldsymbol{J} + \frac{\partial \boldsymbol{E}}{\partial t}$$

All Other Energy

↓

(Electro) Magnetic Energy

MHD scalings: $u^3/\ell c^2$ u/ℓ u/ℓ u/ℓ

$$\frac{\partial}{\partial t}\frac{1}{8\pi}\left(\|\boldsymbol{E}\|^2 + \|\boldsymbol{B}\|^2\right) + \nabla \cdot \frac{c}{4\pi}\boldsymbol{E} \times \boldsymbol{B} = -\boldsymbol{J} \cdot \boldsymbol{E}$$

$$\frac{1}{c^2} \cdot \frac{\partial \boldsymbol{S}}{\partial t} + \nabla \cdot \mathbb{M} = -\rho_e \boldsymbol{E} - \frac{1}{c}\boldsymbol{J} \times \boldsymbol{B}$$

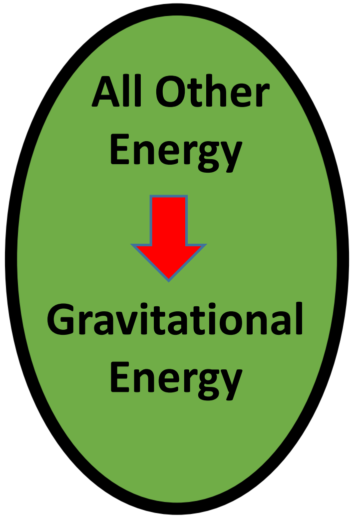
Someone needs to tell us how to determine the electric current and charge density!

Coupling to matter

The Gravitational Field (Review)

$$\nabla^2\Phi = 4\pi G\rho$$

$$\frac{\partial}{\partial t}\rho + \boldsymbol{\nabla}\cdot\rho\boldsymbol{u} = 0$$

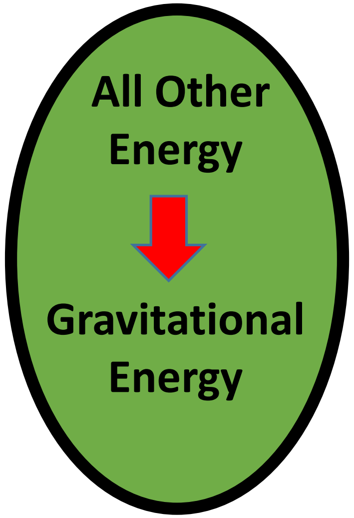


Coupling to matter

$$\frac{\partial}{\partial t}\frac{1}{2}\rho\Phi + \boldsymbol{\nabla}\cdot(\rho\Phi\boldsymbol{u} + \boldsymbol{G}) = \rho\boldsymbol{u}\cdot\boldsymbol{\nabla}\Phi$$

$$\boldsymbol{\nabla}\cdot\mathbb{G} = \rho\boldsymbol{\nabla}\Phi$$

The Gravitational Field (Review)



~~$\nabla^2\Phi = 4\pi G\rho$~~

$$\frac{\partial}{\partial t}\rho + \boldsymbol{\nabla}\cdot\rho\boldsymbol{u} = 0$$

Coupling to matter

$$\frac{\partial}{\partial t}\frac{1}{2}\rho\Phi + \boldsymbol{\nabla}\cdot(\rho\Phi\boldsymbol{u} + \mathbb{G}) = \rho\boldsymbol{u}\cdot\boldsymbol{\nabla}\Phi$$

$$\boldsymbol{\nabla}\cdot\mathbb{G} = \rho\boldsymbol{\nabla}\Phi$$

The Matter (Review)

$$\frac{\partial}{\partial t}\rho + \nabla \cdot \rho \boldsymbol{u} = 0$$

$$\frac{\partial}{\partial t}\boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho}\nabla p - \nabla \Phi + \frac{1}{\rho}\boldsymbol{f}$$

$$\frac{\partial}{\partial t}e + \boldsymbol{u} \cdot \nabla e + \cdot \frac{p}{\rho}\nabla \cdot \boldsymbol{u} = T\dot{s}$$

Note: Thermal conduction and viscous stresses and dissipation can also be accommodated in these terms if desired.

$$\frac{\partial}{\partial t}\frac{1}{2}\rho\|\boldsymbol{u}\|^2 + \nabla \cdot \frac{1}{2}\rho\|\boldsymbol{u}\|^2\,\boldsymbol{u} = -\boldsymbol{u} \cdot \nabla p - \rho \boldsymbol{u} \cdot \nabla \Phi - \boldsymbol{u} \cdot \boldsymbol{f}$$

$$\frac{\partial}{\partial t}\rho e + \nabla \cdot (\rho e + p)\boldsymbol{u} = +\boldsymbol{u} \cdot \nabla p + \rho T \dot{s}$$

$$\frac{\partial}{\partial t}\rho \boldsymbol{u} + \cdot \nabla \cdot (p + \rho \boldsymbol{u}\boldsymbol{u}) = -\rho\,\nabla \Phi + \boldsymbol{f}$$

Someone needs to tell us how to determine the gas pressure!

It remains to determine these terms through full cost accounting!

Momentum Bookkeeping

$$\frac{\partial}{\partial t}\rho\boldsymbol{u} + \cdot\, \boldsymbol{\nabla}\cdot(p + \rho\boldsymbol{uu}) = -\rho\,\boldsymbol{\nabla}\Phi + \boldsymbol{f}$$

$$\boldsymbol{\nabla}\cdot\mathbb{G} = \rho\boldsymbol{\nabla}\Phi$$

$$\frac{1}{c^2}\cdot\frac{\partial \boldsymbol{S}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbb{M} = -\rho_e\,\boldsymbol{E} - \frac{1}{c}\boldsymbol{J}\times\boldsymbol{B}$$

$$\frac{1}{c^2}\cdot\frac{\partial \boldsymbol{F}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbb{P} = \frac{1}{c}\int_0^\infty d\nu\int d\mathbf{n}\,\mathbf{n}[\eta_\nu - \chi_\nu I_\nu]$$

$$\frac{\partial\boldsymbol{\mathfrak{P}}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbb{I} = 0$$

Energy Bookkeeping

$$\frac{\partial}{\partial t}\frac{1}{2}\rho\|\boldsymbol{u}\|^2 + \boldsymbol{\nabla}\cdot\frac{1}{2}\rho\|\boldsymbol{u}\|^2\,\boldsymbol{u} = -\boldsymbol{u}\cdot\boldsymbol{\nabla}p - \rho\boldsymbol{u}\cdot\boldsymbol{\nabla}\Phi + \boldsymbol{u}\cdot\boldsymbol{f}$$

$$\frac{\partial}{\partial t}\rho e + \boldsymbol{\nabla}\cdot(\rho e + p)\boldsymbol{u} = +\boldsymbol{u}\cdot\boldsymbol{\nabla}p + \rho T\dot{s}$$

$$\frac{\partial}{\partial t}\frac{1}{2}\rho\Phi + \boldsymbol{\nabla}\cdot(\rho\Phi\boldsymbol{u} + \boldsymbol{G}) = \rho\boldsymbol{u}\cdot\boldsymbol{\nabla}\Phi$$

$$\frac{\partial}{\partial t}\frac{1}{8\pi}(\|\boldsymbol{E}\|^2 + \|\boldsymbol{B}\|^2) + \boldsymbol{\nabla}\cdot\frac{c}{4\pi}\boldsymbol{E}\times\boldsymbol{B} = -\boldsymbol{J}\cdot\boldsymbol{E}$$

$$\frac{\partial E}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{F} = \int_0^\infty d\nu\int d\mathbf{n}\,[\eta_\nu - \chi_\nu I_\nu]$$

$$\frac{\partial\mathfrak{E}}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{\mathfrak{F}} = 0$$

Full Cost Accounting (Revisited)

$$\boldsymbol{f} = \rho_e \boldsymbol{E} + \frac{1}{c} \boldsymbol{J} \times \boldsymbol{B} - \frac{1}{c} \int_0^\infty d\nu \int d\mathbf{n}\, \mathbf{n} [\eta_\nu - \chi_\nu I_\nu]$$

$$\rho T \dot{s} = \boldsymbol{J} \cdot \boldsymbol{E} - \boldsymbol{u} \cdot \boldsymbol{f} - \int_0^\infty d\nu \int d\mathbf{n}\, [\eta_\nu - \chi_\nu I_\nu]$$

It remains to determine these terms through the geometry of spacetime!

This slide is the ***essential*** objective of RMHD---we have now constructed a set of equations that ***not only*** conserve total energy and momentum, ***but also*** describe how energy and momentum are exchanged between **matter** and the **radiation**, **gravitational** and **electromagnetic** fields!

The "*Golden Rule of RMHD*"

"***Always*** *evaluate interactions between the matter and the classical fields in the* ***co-moving,*** *e.g., rest-frame, of the material!!!"*

but…

*"****Solve*** *your equations in whatever is the most* ***convenient*** *frame of reference for your objectives."*

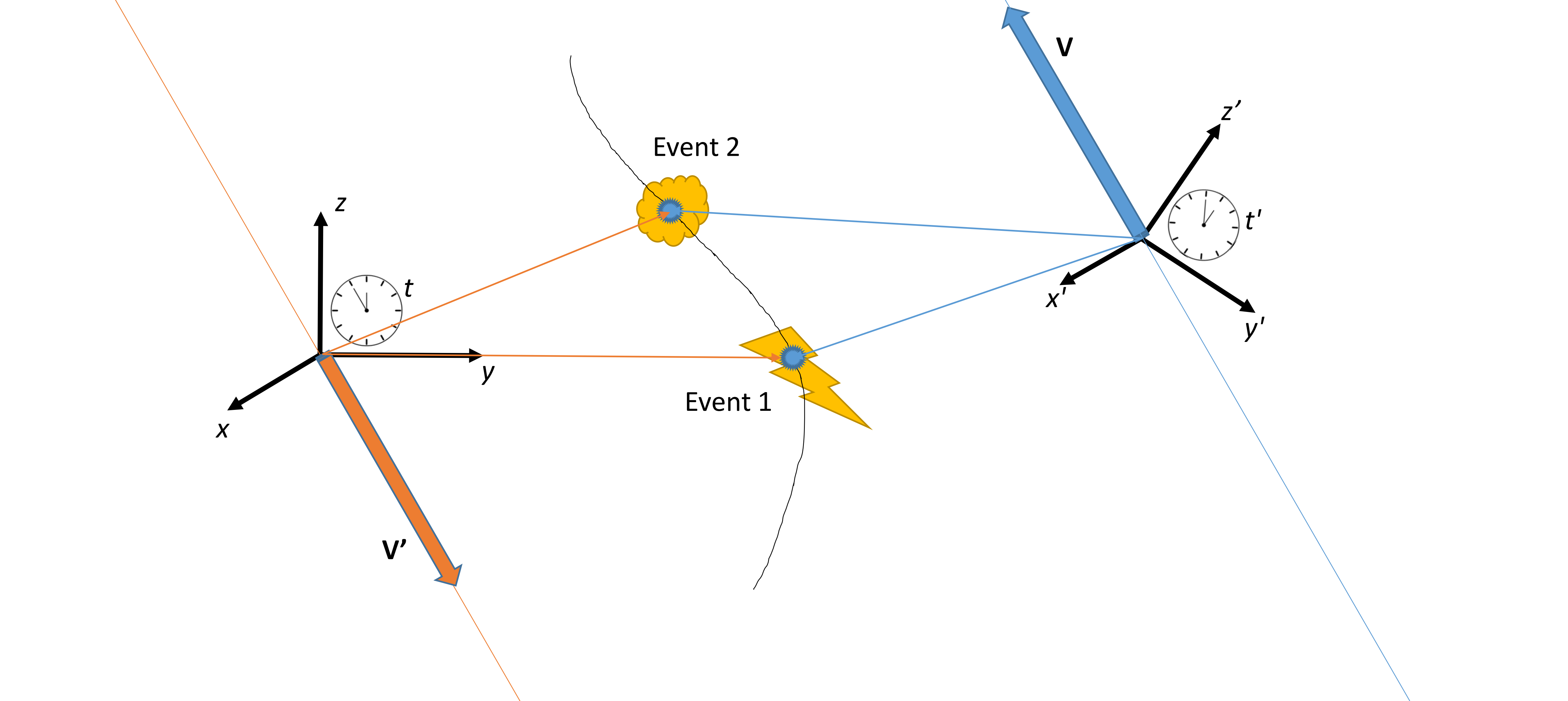
Can I get an *AMEN*!

Corollary to the "*Golden Rule of RMHD*"

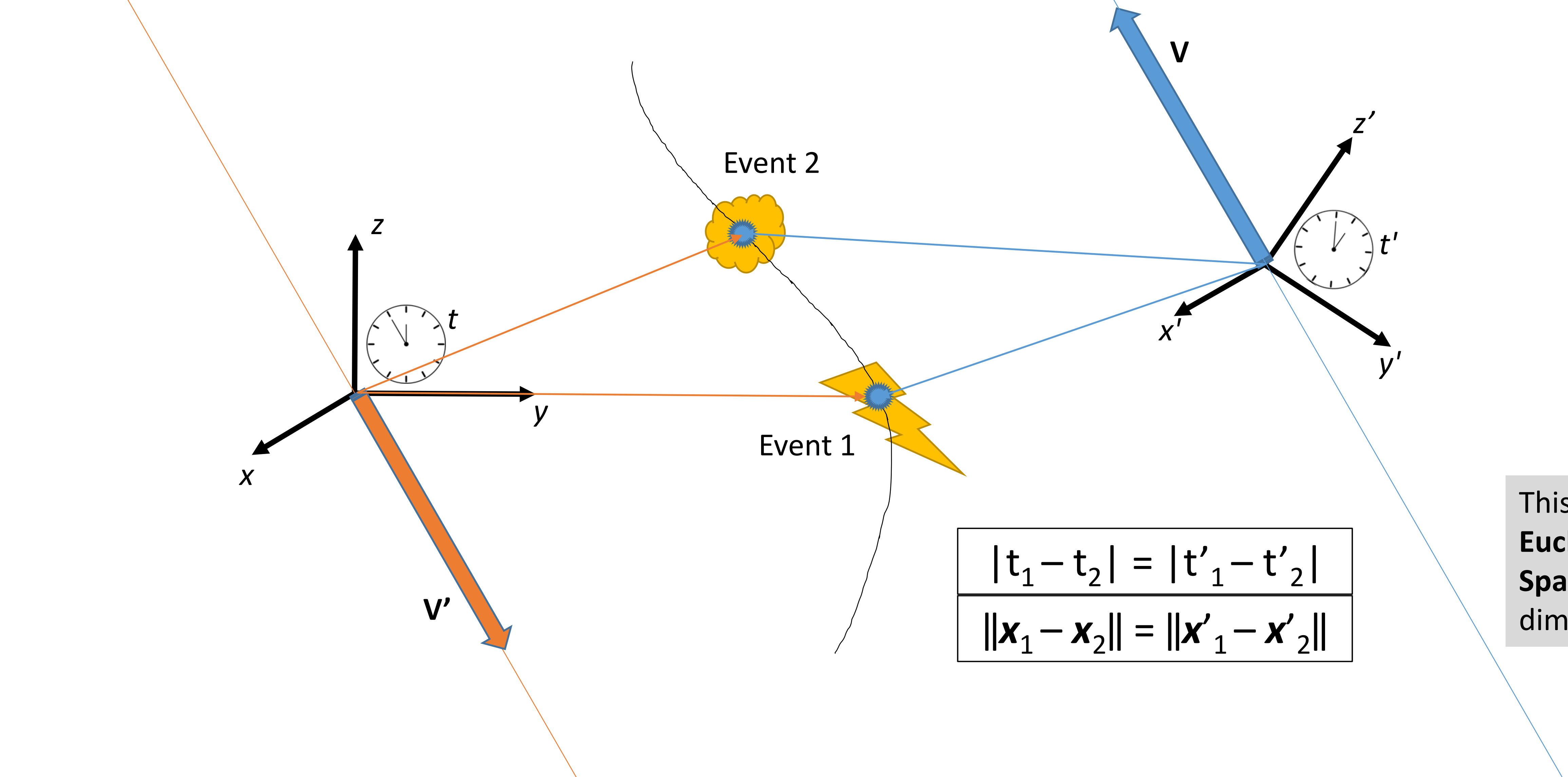
"You better know how to transform coordinates, physical quantities, fields, differential and integral operators (and anything else you can think of) between ***any*** *two frames of reference, under* ***all*** *conditions."*

Abandon hope, all ye who fail to heed the Corollary!

Better Living Through Geometry



Galileo & Newton

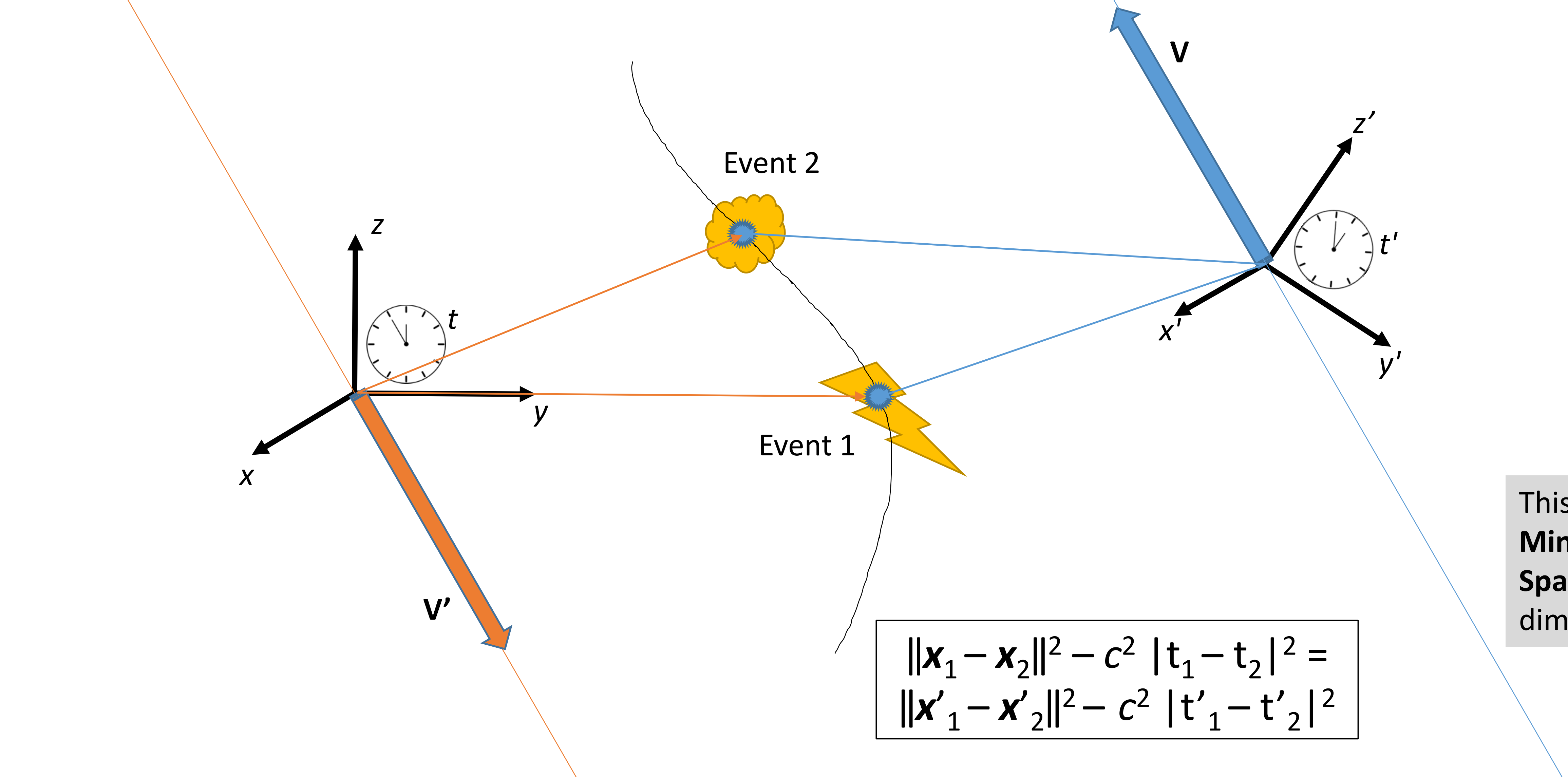


$$|t_1 - t_2| = |t'_1 - t'_2|$$

$$\|\boldsymbol{x}_1 - \boldsymbol{x}_2\| = \|\boldsymbol{x}'_1 - \boldsymbol{x}'_2\|$$

This is a **Euclidean Space** of 4 dimensions.

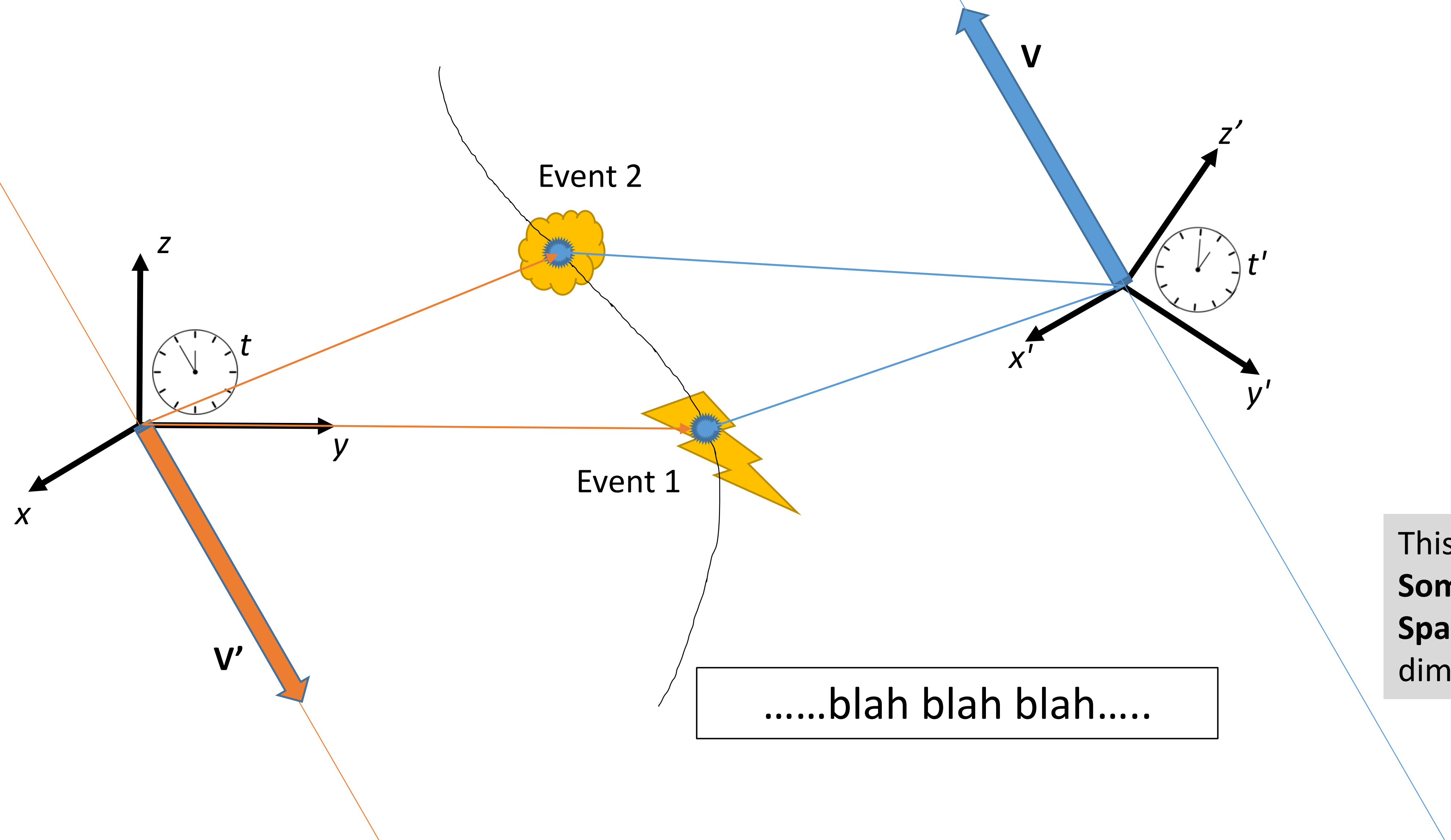
Poincaré & Einstein



$$\|\boldsymbol{x}_1 - \boldsymbol{x}_2\|^2 - c^2\ |t_1 - t_2|^2 =$$
$$\|\boldsymbol{x}'_1 - \boldsymbol{x}'_2\|^2 - c^2\ |t'_1 - t'_2|^2$$

This is a **Minkowski Space** of 4 dimensions.

Invent Your Own Space



This is a **Some Other Space** of 4 dimensions.

Invariants

1 Pick a different origin for marking off time.

3 Pick a different origin for marking off space.

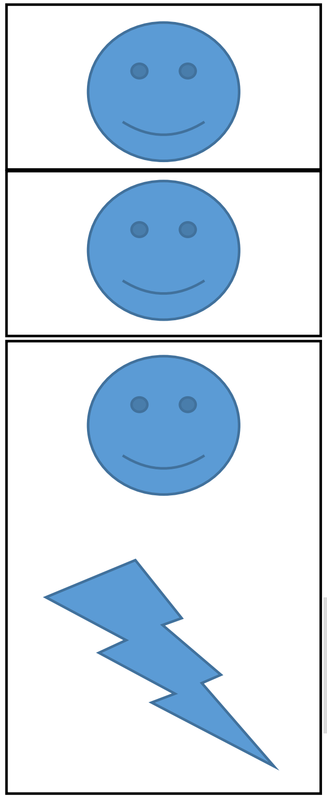
3 Pick a different orientation for your coordinate axes.

3 Move through the space at a constant rectilinear velocity.

10 Parameters

Minkowski Space

$$\|\boldsymbol{x}_1 - \boldsymbol{x}_2\|^2 - c^2\,|t_1 - t_2|^2 = \|\boldsymbol{x}'_1 - \boldsymbol{x}'_2\|^2 - c^2\,|t'_1 - t'_2|^2$$



The Translation Group T(1)

The Translation Group T(3)

The Special Orthogonal Group SO(3)

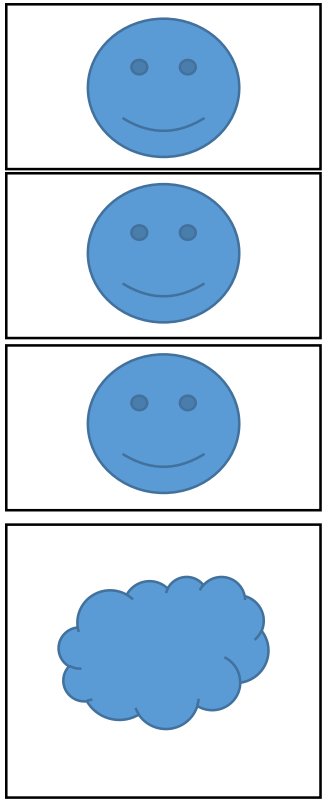
The Lorentz Group

The Poincaré Group

Euclidean Space

$$|t_1 - t_2| = |t'_1 - t'_2|$$

$$\|\boldsymbol{x}_1 - \boldsymbol{x}_2\| = \|\boldsymbol{x}'_1 - \boldsymbol{x}'_2\|$$



Translations

Translations

Proper Rotations

Boosts

The Galilean Group

Group Action

$$\begin{bmatrix} ct' \\ \boldsymbol{x}' \end{bmatrix} \xrightarrow[\mathcal{G}]{\mathcal{L}} \begin{bmatrix} ct'' \\ \boldsymbol{x}'' \end{bmatrix} \xrightarrow[\mathcal{G}^{-1}]{\mathcal{L}^{-1}} \begin{bmatrix} ct' \\ \boldsymbol{x}' \end{bmatrix}$$

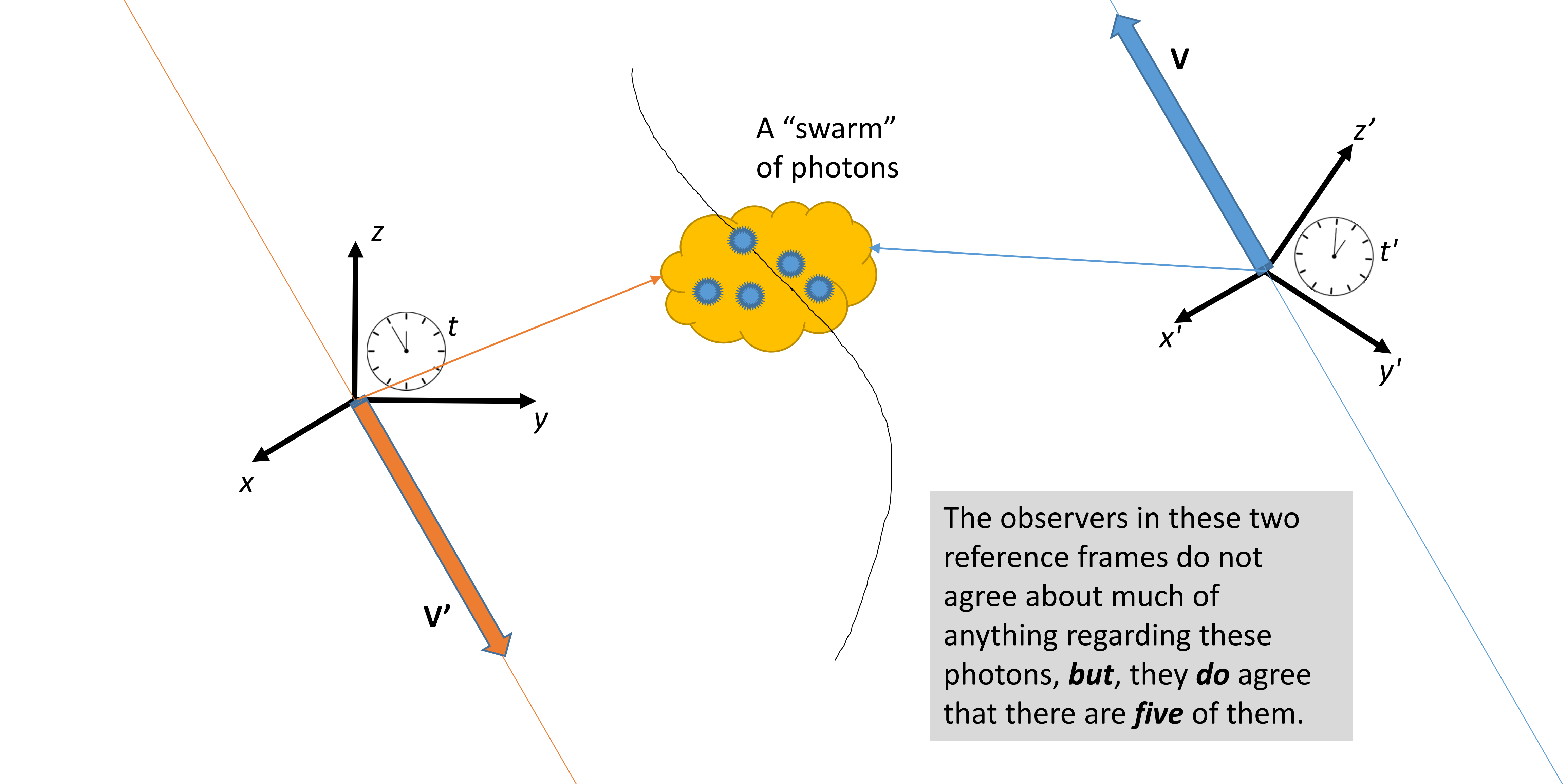
Lorentz Group

Galilean Group

$$\begin{bmatrix} c\rho_e{}' \\ \boldsymbol{J}' \end{bmatrix} \quad \begin{bmatrix} \nu' \\ \nu'\mathbf{n}' \end{bmatrix}$$

Bonus: These objects live in the *tangent space* to each point in the spacetime, and they transform in the same fashion as the spacetime itself!

Count Your Lucky Photons



The observers in these two reference frames do not agree about much of anything regarding these photons, ***but***, they ***do*** agree that there are ***five*** of them.

Lorentz Transformations of the Radiation

$$\frac{I_\nu}{\nu^3} = \frac{I'_{\nu'}}{\nu'^3}$$

$$\frac{\chi_\nu}{\nu'} = \frac{\chi'_{\nu'}}{\nu}$$

$$\frac{\eta_\nu}{\nu^2} = \frac{\eta'_{\nu'}}{\nu'^2}$$

$$\nu = \Gamma_v\, \nu' \left(1 + \frac{1}{c}\mathbf{n}' \cdot \mathbf{v}\right)$$

$$\nu' = \Gamma_v\, \nu \left(1 - \frac{1}{c}\mathbf{n} \cdot \mathbf{v}\right)$$

The corresponding equations for $\mathbf{n}$ and $\mathbf{n}$’ are more complicated, but, fortunately, we don’t need them here.

If you remember only ***one*** thing from this lecture, ***this*** would be it.

Gray Approximation in the *Co-Moving* Frame

$$\nu = \Gamma_v \, \nu' \left(1 + \frac{1}{c}\mathbf{n}' \cdot \mathbf{v}\right)$$

$$\nu' = \Gamma_v \, \nu \left(1 - \frac{1}{c}\mathbf{n} \cdot \mathbf{v}\right)$$

$$\frac{\chi_\nu}{\nu'} = \frac{\kappa}{\nu}$$

$$\frac{\eta_\nu}{\nu^2} = \frac{\kappa B_{\nu'}}{\nu'^2}$$

A ***constant***, frequency-independent opacity, and the ***isotropic*** Planck Function (but frequency dependent) constitutes the "gray atmosphere" approximation in the co-moving frame.

Notice that in the laboratory frame the emissivity and the opacity are ***not*** isotropic because of the Doppler shifted frequency!

Gray Approximation in the *Laboratory* Frame

$$\eta_\nu = \kappa \left[\left(1 + \frac{3}{c} \mathbf{n}\cdot\mathbf{v}\right) B_\nu - \frac{1}{c} \mathbf{n}\cdot\mathbf{v} \frac{\partial}{\partial \nu} \nu B_\nu \right] + \cdots$$

$$\chi_\nu = \kappa \left(1 - \frac{1}{c} \mathbf{n}\cdot\mathbf{v}\right) + \cdots$$

We can now carry out the two integrals we need to describe the exchange of energy and momentum between the material and the radiation field in the ***laboratory frame***.

$$\frac{\partial E}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{F} = \int_0^\infty d\nu \int d\mathbf{n} \left[\eta_\nu - \chi_\nu I_\nu \right]$$

Gray Approximation in the *Laboratory* Frame

$$\eta_\nu = \kappa \left[\left(1 + \frac{3}{c}\mathbf{n}\cdot\mathbf{v}\right) B_\nu - \frac{1}{c}\mathbf{n}\cdot\mathbf{v}\frac{\partial}{\partial\nu}\nu B_\nu\right] + \cdots$$

$$\chi_\nu = \kappa \left(1 - \frac{1}{c}\mathbf{n}\cdot\mathbf{v}\right) + \cdots$$

We can now carry out the two integrals we need to describe the exchange of energy and momentum between the material and the radiation field in the ***laboratory frame***.

$$\frac{\partial E}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{F} = \kappa\left[4\sigma_R T^4 - cE\right] + \kappa\frac{1}{c}\boldsymbol{u}\cdot\boldsymbol{F} + \cdots$$

Gray Approximation in the *Laboratory* Frame

$$\eta_\nu = \kappa \left[\left(1 + \frac{3}{c}\,\mathbf{n}\cdot\mathbf{v}\right) B_\nu - \frac{1}{c}\,\mathbf{n}\cdot\mathbf{v}\,\frac{\partial}{\partial\nu}\,\nu B_\nu\right] + \cdots$$

$$\chi_\nu = \kappa \left(1 - \frac{1}{c}\,\mathbf{n}\cdot\mathbf{v}\right) + \cdots$$

We can now carry out the two integrals we need to describe the exchange of energy and momentum between the material and the radiation field in the ***laboratory frame***.

$$\frac{1}{c^2}\cdot\frac{\partial \boldsymbol{F}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbb{P} = \frac{1}{c}\int_0^\infty d\nu \int d\mathbf{n}\,\mathbf{n}\left[\eta_\nu - \chi_\nu I_\nu\right]$$

Gray Approximation in the *Laboratory* Frame

$$\eta_\nu = \kappa \left[\left(1 + \frac{3}{c}\mathbf{n}\cdot\mathbf{v}\right) B_\nu - \frac{1}{c}\mathbf{n}\cdot\mathbf{v}\frac{\partial}{\partial\nu}\nu B_\nu\right] + \cdots$$

$$\chi_\nu = \kappa \left(1 - \frac{1}{c}\mathbf{n}\cdot\mathbf{v}\right) + \cdots$$

We can now carry out the two integrals we need to describe the exchange of energy and momentum between the material and the radiation field in the ***laboratory frame***.

$$\frac{1}{c^2}\cdot\frac{\partial \boldsymbol{F}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbb{P} = -\frac{\kappa}{c}\left[\boldsymbol{F} - \boldsymbol{u}\left\{\frac{4\sigma_R}{c}T^4 + \mathbb{P}\right\}\right] + \cdots$$

Thomson Scattering in the *Co-Moving* Frame

$$\frac{I_\nu}{\nu^3} = \frac{I'_{\nu'}}{\nu'^3}$$

$$\frac{\chi_\nu}{\nu'} = \frac{\sigma}{\nu}$$

$$\frac{\eta_\nu}{\nu^2} = \frac{\sigma J'_{\nu'}}{\nu'^2}$$

$$\nu = \Gamma_v \, \nu' \left(1 + \frac{1}{c}\mathbf{n}' \cdot \mathbf{v}\right)$$

$$\nu' = \Gamma_v \, \nu \left(1 - \frac{1}{c}\mathbf{n} \cdot \mathbf{v}\right)$$

A ***constant***, frequency-independent opacity, and the ***mean intensity*** (but frequency dependent) as source function constitutes the limit of pure Thomson Scattering off electrons in the co-moving frame.

Notice that in the laboratory frame the emissivity and the opacity are ***not*** isotropic because of the Doppler shifted frequency!

Thomson Scattering in the *Co-Moving* Frame

$$\frac{I_\nu}{\nu^3} = \frac{I'_{\nu'}}{\nu'^3}$$

$$\frac{\chi_\nu}{\nu'} = \frac{\sigma}{\nu}$$

$$\frac{\eta_\nu}{\nu^2} = \frac{\sigma J'_{\nu'}}{\nu'^2}$$

$$\nu = \Gamma_V\, \nu' \left(1 + \frac{1}{c}\, \mathbf{n}' \cdot \mathbf{v}\right)$$

$$\nu' = \Gamma_V\, \nu \left(1 - \frac{1}{c}\, \mathbf{n} \cdot \mathbf{v}\right)$$

Your HOMEWORK Assignment for Thursday!

$$\frac{\partial E}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{F} = \int_0^\infty d\nu \int d\mathbf{n}\, [\eta_\nu - \chi_\nu I_\nu]$$

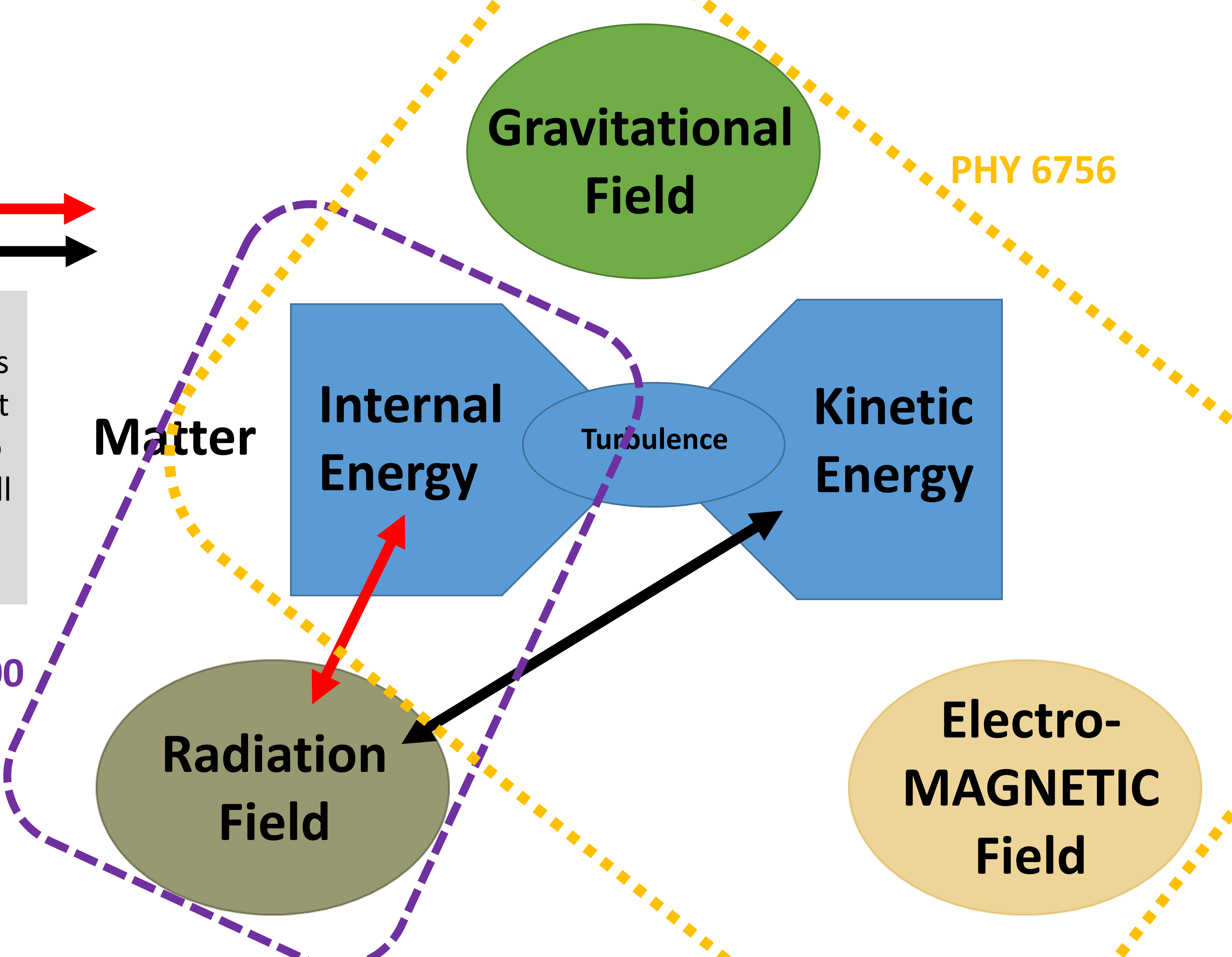
$$\frac{1}{c^2} \cdot \frac{\partial \boldsymbol{F}}{\partial t} + \boldsymbol{\nabla} \cdot \mathbb{P} = \frac{1}{c} \int_0^\infty d\nu \int d\mathbf{n}\, \mathbf{n} [\eta_\nu - \chi_\nu I_\nu]$$

Summary

Entropy Production →

Adiabatic (Reversible) →

We worked ***really*** hard to get these two arrows ***correct*** to order u/c ---it is actually quite easy to get them wrong, as well as everything else that follows…

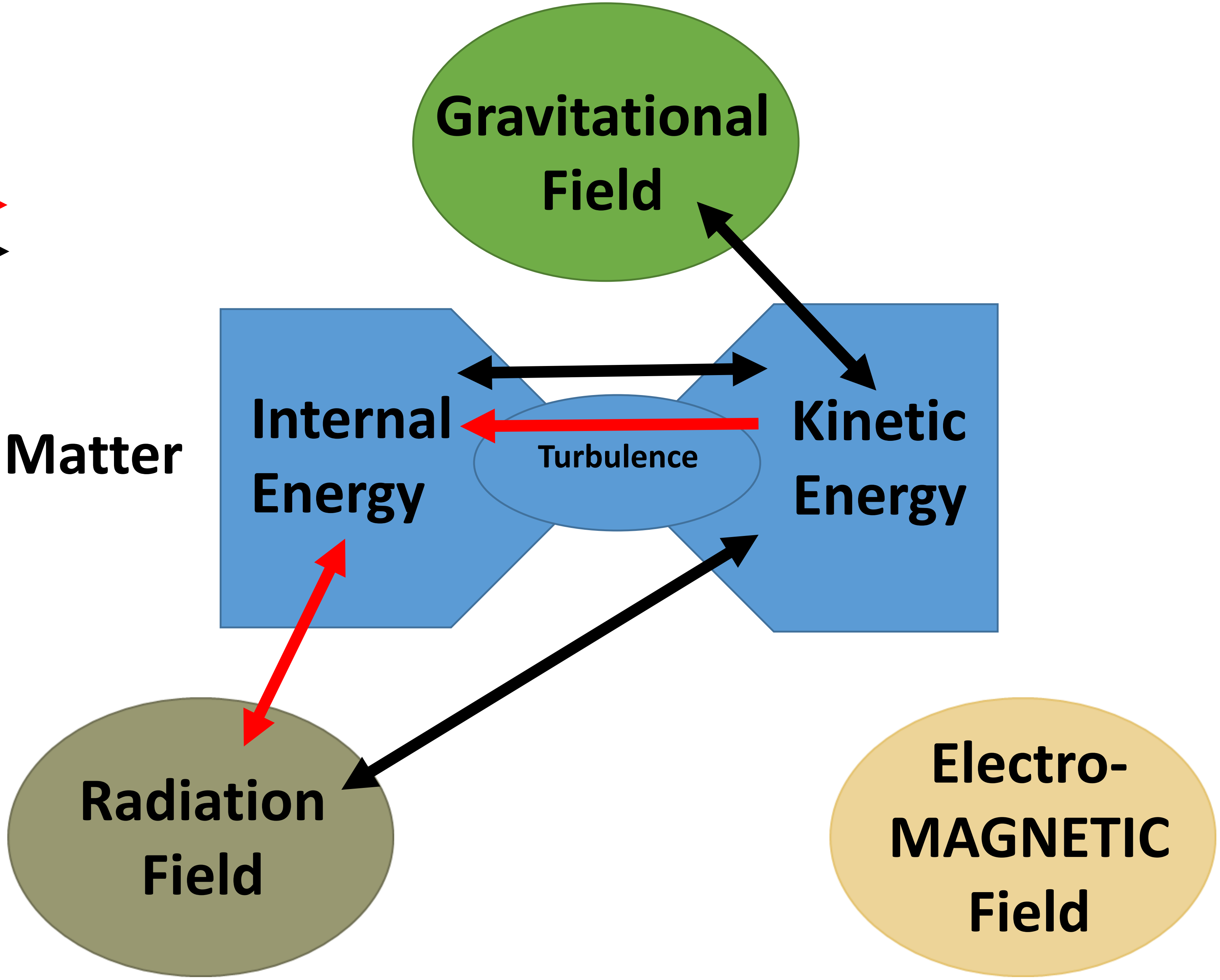


Next Lecture

Entropy Production (red arrow)

Adiabatic (Reversible) (black arrow)

Spherically-symmetric winds and accretion flows!



Merci! À Bientôt.

Gosh, I sure hope I am in the rest frame of the fluid…