A NEW AVALANCHE MODEL FOR SOLAR FLARES

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ABSTRACT

Assuming that solar flares are avalanches of many small reconnection events and that solar coronal magnetic field is in a self-organized critical state we develop a new generation of SOC models. We construct a 2D lattice formed of parallel 'field lines' that are randomly deformed, leading to the development of tangential discontinuities in the field lines. This simple device is a working scheme that will lead us to a more physically realistic SOC numerical model of solar flares.

Key words: Sun: flares - Sun: magnetic fields.

1. INTRODUCTION

The solar corona is formed by a magnetized plasma characterized by temperatures of the order of 2 millions of degrees; when a solar eruption takes place its temperature can be $\sim 10^7 K$ locally.
Analysis of many hundreds of thousands of flares reconstructed from UV, EUV and X-ray observations have revealed the remarkable fact that the frequency distribution of the energy released by flares has the form of a tight power-law, spanning at least eight decades in flare energy, implying self-similarity, and thus the lack of a 'typical' scale for the flaring process (Dennis, 1985).
In the 1980's E. N. Parker (Parker, 1983) suggested a physical mechanism for coronal heating via numerous small scale reconnection events: the so-called *nano-eruptions*. His physical picture consists of a magnetic structure embedded in the solar corona such as a coronal loop. The loop is anchored to the photosphere where its footpoints are subject to random horizontal fluid motions associated with photospheric manifestations of convection and granulation. At photospheric levels the energy density of the plasma greatly exceeds that of the magnetic field, so the field cannot resist the motion of the electrically conducting fluid and, consequently are shuffled around as they each execute a form of random walk. The high electrical conductivity of the coronal gas implies that the magnetic field is frozen-in to it thus the subsequent dynamical relaxation within the loops results in a complex, tangled magnetic field that is force free everywhere but in numerous small electrical current sheet in regions where the field lines are twisted or braided around one another. These electrical currents build up to the point that the onset of magnetic reconnection becomes inevitable, leading to energy release in the corona.
Parker's estimation for the energy released in such events is $10^{24} erg$ (Parker, 1988) which is nine orders of magnitude smaller than the energy released by large flares.
The magnetic field reconfiguration taking place in the vicinity of reconnecting current sheets will alter the physical conditions around neighboring currents sheets, and may lead to further reconnection events at some of these sheets, and so on, causing and avalanche of reconnection events cascading throughout the tangled magnetic structure.
Parker's model contains all the ingredients necessary to lead to self-organized criticality, and inspired Lu & Hamilton (Lu & Hamilton, 1991) to suggest that the solar corona magnetic field is in a state of self-organized criticality, that flares are simply the energy collectively released by an avalanche of reconnection events.
The sandpile is the most typical example of a self-organized critical system (see details on this exemplar system in (Bak, Tang & Wiesenfeld) and (Bak, 1996)). As sand is added to a sandpile the slope steepens until it reaches a critical angle beyond which further addition of sand causes avalanches in order to readjust the local shape of the pile. The critical state is insensitive to the initial conditions, does not need the tuning of any parameter and the critical state is an attractor of the dynamics. Also central to SOC systems is that they are interaction-dominated, in the sense that the dynamical behavior is an emergent property of the simple interaction between many degrees of freedom, and that the avalanching process has no preferred scale for the release of energy.
In section 2 we present in detail the new avalanche model for solar flares that we have developed.

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Section 3 is devoted to the discussion of some of the preliminary results obtained. Finally in section 4 we summarize our main results.

2. THE NEW SOC MODEL

Most of the avalanches models for solar flares used the basic formulation developed by Lu & Hamilton in 1991 (a basic introduction to these models can be found in (Charbonneau et al., 2001)). In this section we will describe the elements and the dynamics of the system we used to model solar flares as avalanches.

2.1. The lattice and the driving mechanism

We define a $2\,D$ lattice of size $N \times N$ as a network of vertically interconnected nodes with periodic boundary conditions in the horizontal direction. Thus the lattice can be thought as a 'straightened' coronal loop.
We label each node of the $2\,D$ lattice using a vectorial index $\mathbf{k} = (i, j)$ as shown in Figure 1.
At the initial time each set of vertically interconnected nodes is defined to be magnetic field line and labeled under the index i. The length of each magnetic field line will be $l_i(0) = N - 1$. The system is then perturbed with the successive displacement of randomly chosen nodes (one per step) leading to a larger field line configuration.
The direction of the displacement (whether it is right or left) is chosen randomly and for the amplitude we work both with symmetrical and anti symmetrical selections.

The magnetic intensity of each field line is related to the length $l(t)$ of the magnetic field lines by means of the flux conservation condition so $B_i(t) \propto l_i(t)$.
Using this relation the lattice magnetic energy (E) is:

$$E(t) = \sum_i l_i(t)^2 \qquad (1)$$

where the sum over i stands for all the columns of the lattice.
The energy of a $N \times N$ lattice of initially straight field lines is:

$$E(0) = N\,(N - 1) \qquad (2)$$

2.2. The stability criterion

As the nodes are displaced and two or more lines cross at the same node it is possible to calculate the angle formed between a pair of field lines. The current density can be calculated using Maxwell's equation: $\boldsymbol{j} = \frac{c}{4\pi}\,\nabla \times \boldsymbol{B}$. The stress between two lines of magnetic field will determine a greater value for the current density. Keeping this in mind a node is deemed unstable if the local value of the angle formed between two field lines exceeds some fixed threshold, just as in Parker's (Parker, 1983) physical scenario.

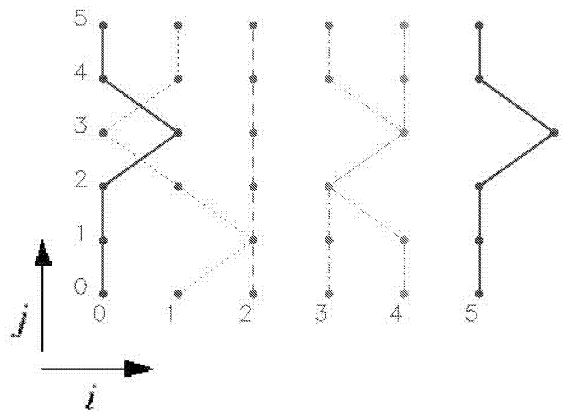

Figure 1: Detail of a small lattice showing the main characteristics of the model.
Three angles are displayed its values are: $\theta_{(2,1)} = \frac{\pi}{2}$ and $\theta_{(3,2)} = \theta_{(4,3)} = \frac{\pi}{4}$.

2.3. The redistribution rule

Once a node is deemed unstable a procedure is needed to restore stability. The procedure we have chosen as our redistribution rule consist in displacing one of the field lines involved in the unstable angle formation in such a way that its length will be shortened locally. In this case the displacements are of amplitude one.
It is possible that the re-located field line may be involved in the formation of another angle (but with a different field line and on a different node), so that this new node may also become unstable. In that case the redistribution rule is applied again. The sequence of these redistribution events is the model's realization of an avalanche.
Whenever a field line node is re-located some magnetic energy is released. The total amount of magnetic energy released at each iteration will be directly related to the shortening of the magnetic field line because of the application of the redistribution rule, and is calculated using equation (1) as the difference between the lattice magnetic energy before and after the redistribution process.
It is important to notice here that when an avalanche is taking place no driving is exerted over the lattice,

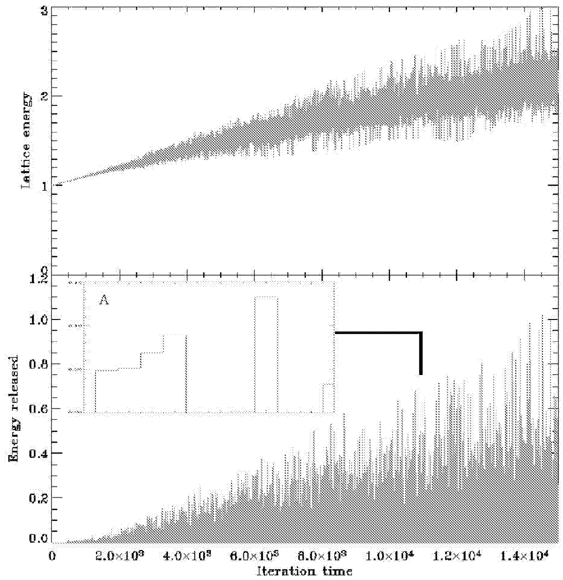

Figure 2: Time series of lattice energy for a lattice of size 128^2 symmetrically driven (up) and energy released in avalanches for the same lattice (down). The inset (A) shows a small portion of energy released time series and illustrates the discrete nature of the releasing energy process. Both quantities have been normalized using the initial magnetic energy of the lattice ($E(0)$).

i.e. random displacements are only introduced whenever there is no unstable nodes in the lattice.

3. PRELIMINARY RESULTS

In a $2\,D$ lattice containing $N \times N$ nodes the maximum angle between two field lines may be:

$$\theta = \lim_{N\to\infty} 4\,tan^{-1}\left(\frac{N}{2}\right) = 2\pi$$

(3)

taken this into account we have chosen our angle threshold take $\frac{1}{3}$ of that value.

We performed simulations for different sizes of the $2\,D$ lattice. On Figure 2 we present the results obtained for lattices of 128×128 integrated over $1.5\,10^4$ iterations and symmetrically driven. The upper panel shows the time series of the lattice energy and the lower panel shows the energy released

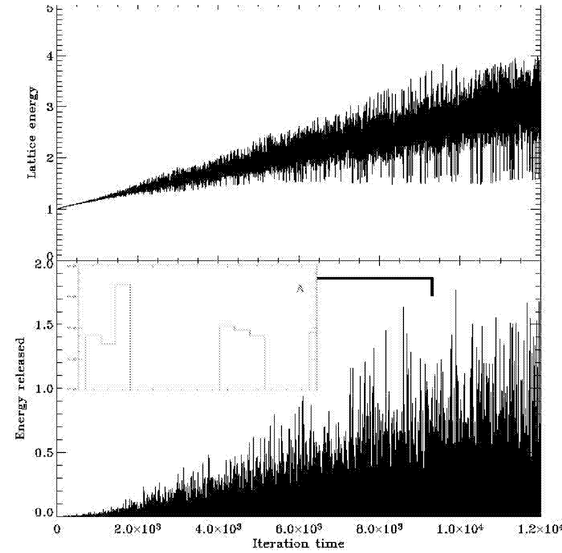

Figure 3: Time series of lattice energy for a lattice of size 128^2 non-symmetrically driven (up) and energy released in avalanches for the same lattice (down). The inset (A) shows a small portion of energy released time series and illustrates the discrete nature of the releasing energy process. We have normalized using the initial magnetic energy of the lattice ($E(0)$).

by the system.

In the first iterations we observe mainly the growth of magnetic energy due to the added perturbation that enhances the magnetic field lines. As time goes on the first avalanches appear, becoming more and more frequent and of greater magnitude as the system evolves further in time.

On Figure 3 we present the same series as in Figure 2 but in this case we have driven in a non-symmetrical way (amplitude 3 to the right and amplitude 1 to the left) and we have iterated over $1.2\,10^4$ time steps.
Although the lattice energy and the energy released are of the same order for both simulations the non-symmetrically driven case presents an enhancement of both the lattice energy and the energy released.
The energy released is spanned in both cases over a wide range since the lowest normalized energy released is $\sim 1.210^{-7}$ and the greatest is ~ 1.02 for the symmetrically driven simulation and the same values for the non-symmetrically driven system are $\sim 1.210^{-7}$ and ~ 1.7 respectively.
This is a very important result as we have found that the energy released values extend over seven order of magnitude thus showing one of the main char-

acteristics a SOC model should be able to reproduce.

4. DISCUSSION

We have constructed a new avalanche model for solar flares which (at least at this early stage of development) exhibits the desired features of the SOC states, and in which the different elements of the numerical model are associated to physical quantities with the less possible ambiguity (for discussion on the different interpretations of other models see (Lu et al, 1993), (Isliker, Anastasiadis & Vlahos), (Isliker et al, 1998)).
Although the instability threshold is small we were able to find that the size of the avalanches vary over a great range. This will allow us to begin the analysis of the statistical distributions of the typical parameters that characterize avalanches, such as the integrated energy, the peak energy released, the avalanche duration and inter-avalanche waiting time.
Many of the avalanches models developed in the past were shown to be sensitive to the size of the lattice and insensitive to the value of the instability threshold and produced a SOC state when the perturbation was not symmetrical (many of this results are reported in (Charbonneau et al., 2001)). Our model has proven to produce avalanches when the driving exerted is applied symmetrically and non-symmetrically. In the future we shall be testing the behavior of our new avalanche model when the driving mechanism parameters are modified.

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