Solar Flare Models and Data Assimilation

Eric Bélanger

belanger@ASTRO.UMontreal.CA

Thesis Directors :

Alain Vincent

Paul Charbonneau

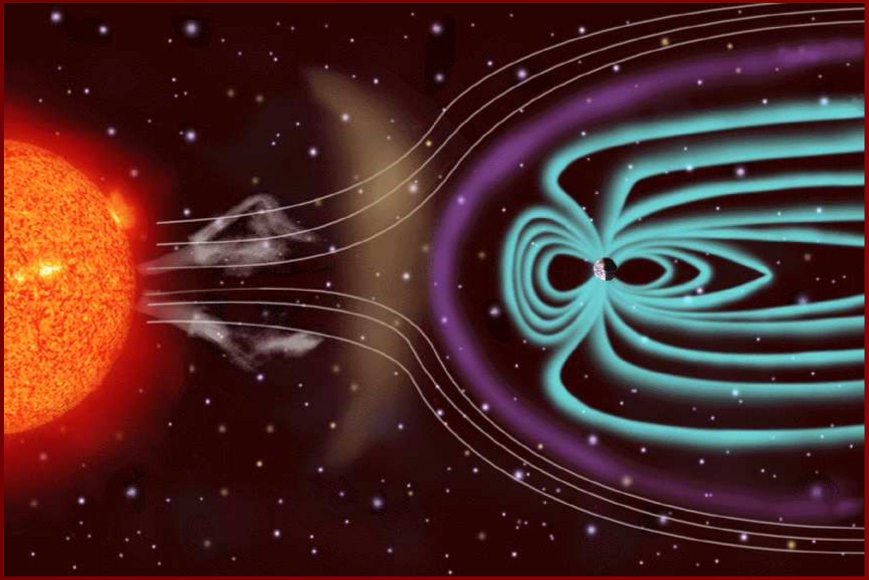
Département de physique

Université de Montréal

Summary

- Sun-Earth connection
- Satellite observation
- Solar flare theory
- Solar flare models
- Data assimilation methods
- Results

The Sun-Earth connection



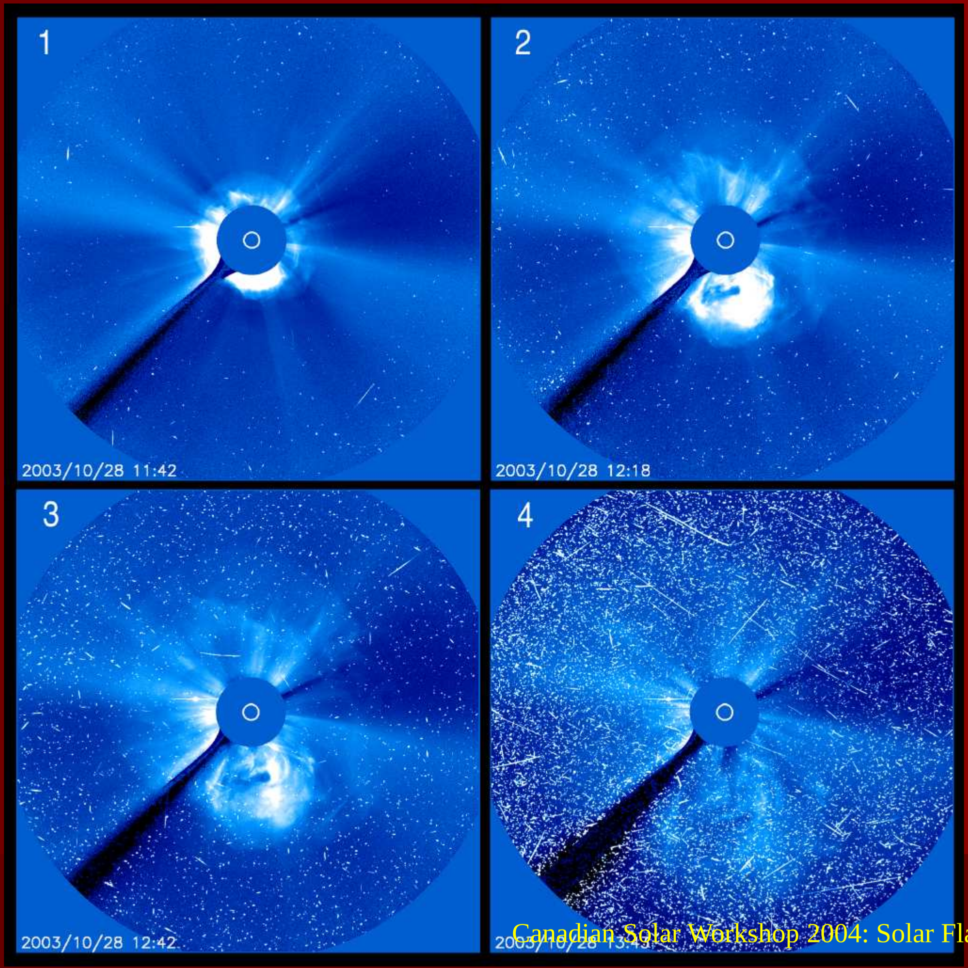
- Equilibrium state between the magnetosphere and the solar wind
- Coronal mass ejection caused by solar flare
- Perturbed equilibrium : $\frac{d\vec{B}}{dt} \neq 0$
- Induction of electric field : $\frac{d\vec{B}}{dt} = -\vec{\nabla} \times \vec{E}$
- Electrical currents flowing in the ionosphere and in the ground

Transition Region and Coronal Explorer (TRACE)

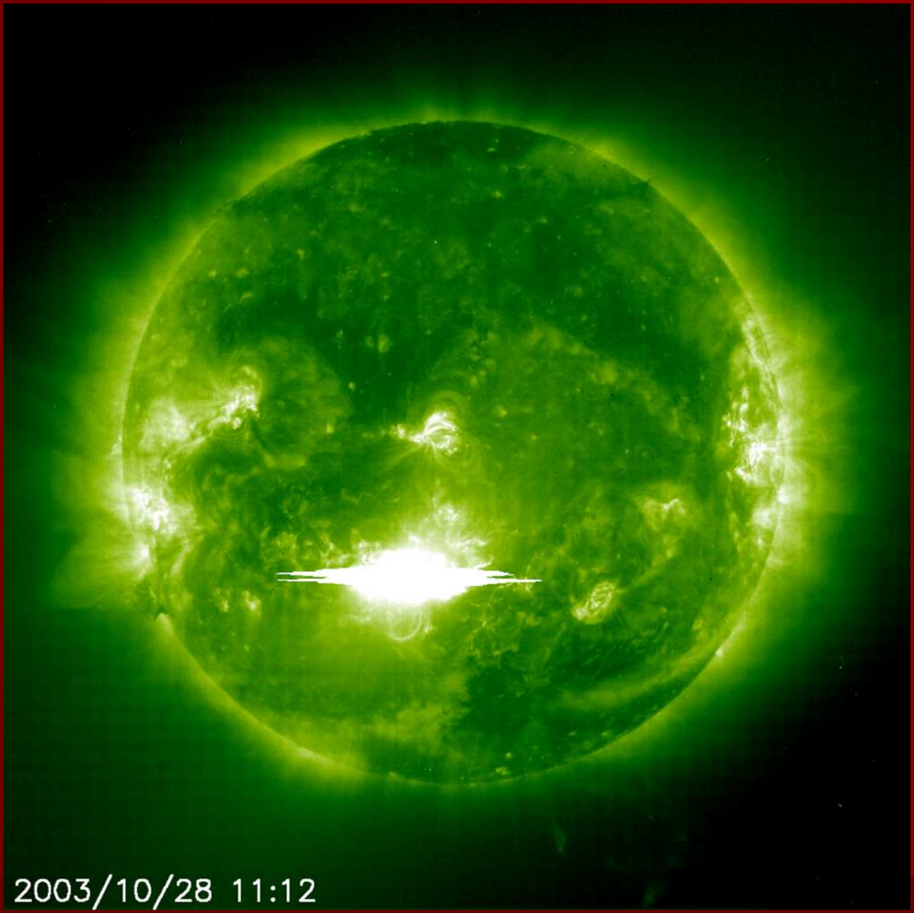
Solar & Heliospheric Observatory (SOHO)

The 28 October (2003) flare

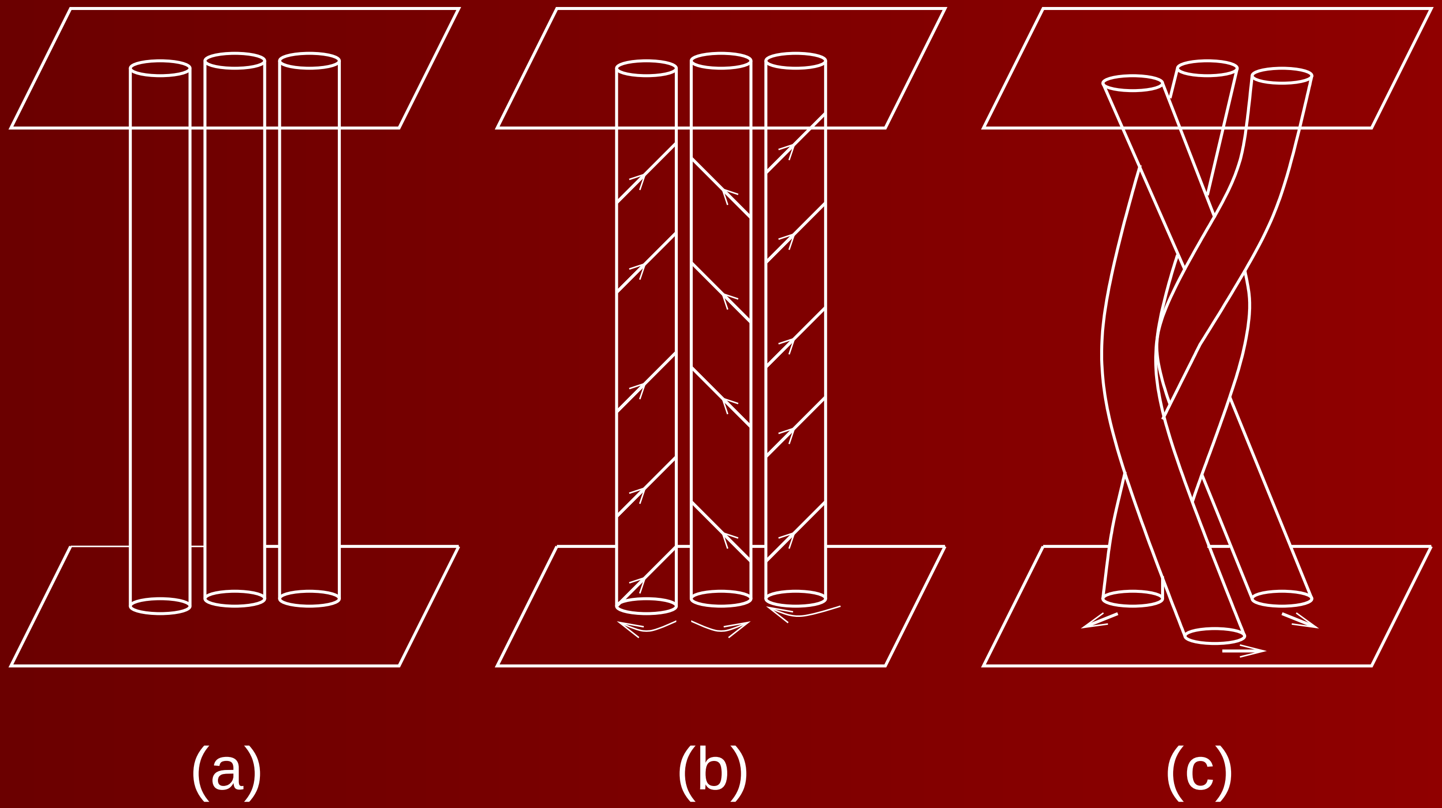
- Large Angle and Spectrometric Coronograph (LASCO)



- Extreme Ultraviolet Imaging Telescope (EIT)
  - Fe XII (195 Å) emission



Flux tubes twisting (Parker, 1983)



Power law (Aschwanden et al., 2000)

The energy frequency distribution :

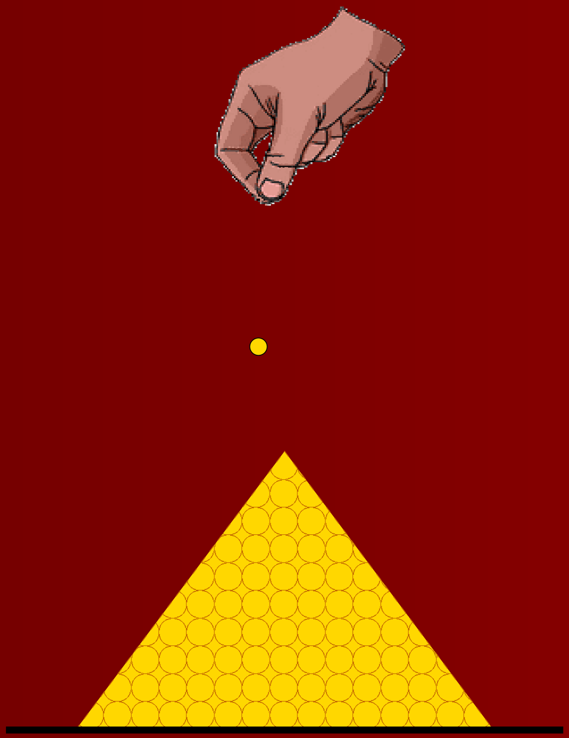
$$f(E) = f_0 E^{-\alpha}, \qquad \alpha > 0$$

Total energy :

$$\begin{aligned} E_{tot} &= \int_{E_{min}}^{E_{max}} f(E) E \mathrm{d}E \\ &= f_0 \left(\frac{E^{2-\alpha}}{2-\alpha} \right) \Bigg|_{E_{min}}^{E_{max}}, \qquad \alpha \neq 2 \end{aligned}$$

- $\alpha > 2 \Rightarrow$ small flares are dominant (for coronal heating)
- $\alpha < 2 \Rightarrow$ large flares are dominant (for coronal heating)

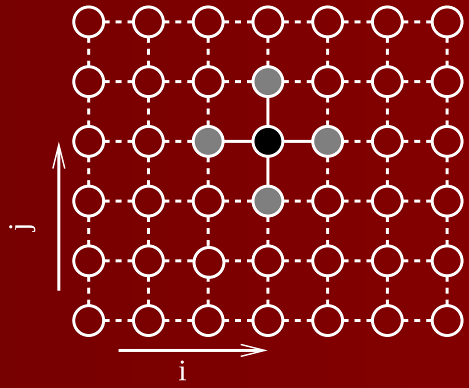
Self-organized criticality



- an open system with slow external forcing
- a self-stabilizing instability threshold
- a local redistribution of a dynamic variable

Avalanche model

- Lattice of metastable sites (Charbonneau et al., 2001)



- Stability criterion :

$$\Delta A_{i,j}^{n} \equiv A_{i,j}^{n} - \frac{1}{2D} \sum_{\text{neighbours}} A_{\text{neighbours}}^{n}$$

- Redistribution of A :

$$A_{i,j}^{n+1} = A_{i,j}^{n} - \frac{2D}{2D+1} \Delta A_{i,j}^{n}$$
$$A_{i\pm1,j\pm1}^{n+1} = A_{i\pm1,j\pm1}^{n} + \frac{1}{2D+1} \Delta A_{i,j}^{n}$$

- Continuous avalanche model :

$$\frac{\partial A}{\partial t} = -\frac{\partial^2}{\partial x^2}\left(\nu(A^2_{xx})\frac{\partial^2 A}{\partial x^2}\right) + F_R$$

with :

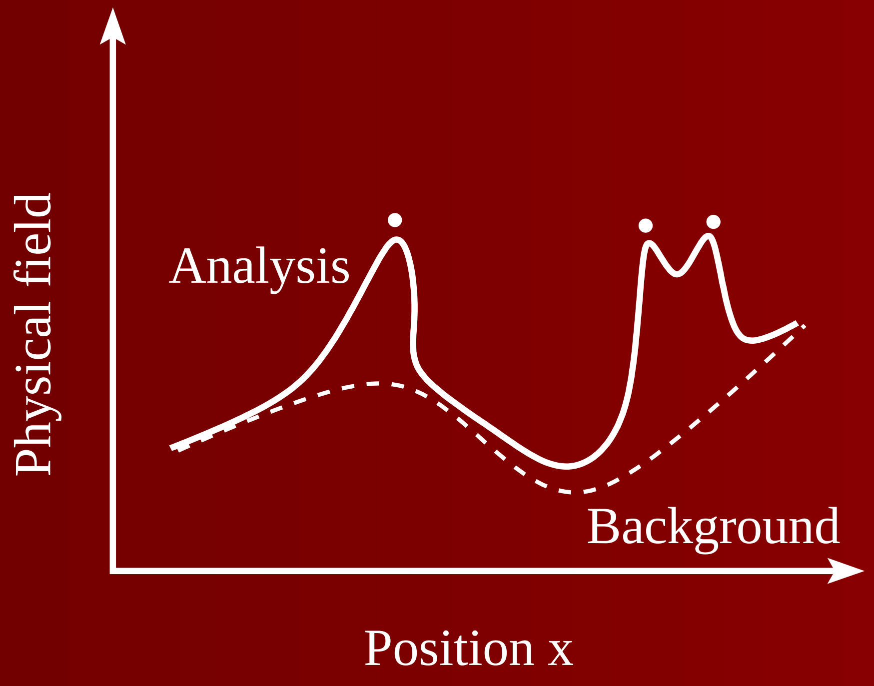
$$\nu(A^2_{xx}) = \begin{cases} \nu_a & \text{if } \Delta A^2 > A_c^2 \\ 0 & \text{else} \end{cases}$$

Data Assimilation

So far, we have considered the use of the observational satellites and of the theoretical/numerical models to understand the complex phenomena of the solar flares in order to make forecasts. Data assimilation is an efficient technique to incorporate observations in numerical models.

Successive corrections methods

$$X_a(i) = X_b(i) + w(i)E(i)$$



Statistical methods

These methods are based on the combination of the characteristic errors of the observations and models to minimize them statistically.

- Kalman filter - The statistics of the model's errors are updated
- Optimal interpolation - The statistics of the model's errors are kept constant

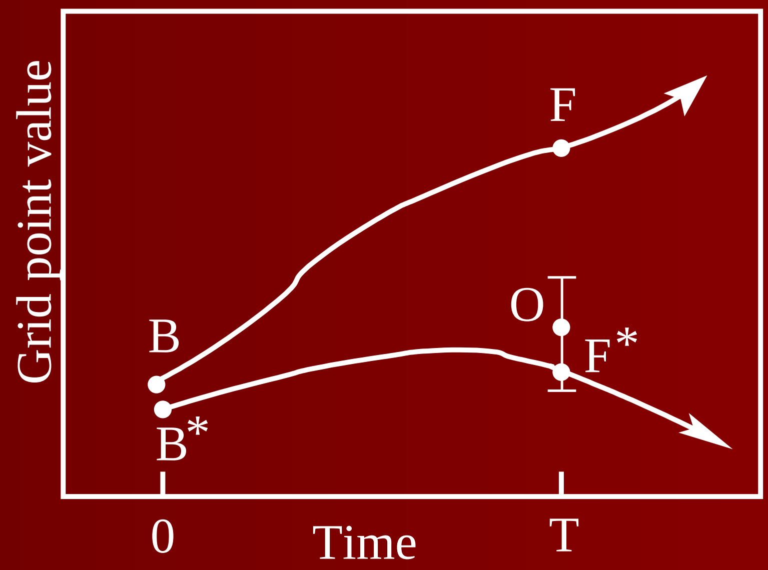
Example of Kalman filter :

- X_1 Model
- X_2 Observation

$$X_a = X_1 + K(X_2 - X_1)$$

$$K = \frac{1}{\sigma_2^2}\left[\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right]^{-1}$$

Variational methods (Kalnay, 2003)



- B : estimation of the system initial state
- F : forecast at time T
- O : observation
- B^* : correction of the initial state estimation
- F^* : new forecast at time T

Lyapunov exponents

- In a time interval Δt, a small initial perturbation (error) $\epsilon(\Delta t)$ grows exponentially :

$$\epsilon(\Delta t) \propto e^{\Lambda \Delta t}$$

  where Λ is a Lyapunov exponent.
- $\Lambda \leq 0 \Longrightarrow$ stable system
- $\Lambda > 0 \Longrightarrow$ unstable system

The cost function

- Generally, the cost function is written as :

$$\mathcal{J} = \int_0^T \int_\Omega f(\overrightarrow{\Psi}, \vec{x}, t)\, d\vec{x}\, dt$$

- More precisely :

$$\mathcal{J} = \frac{1}{2} \int_0^T \int_\Omega (A - A_{\mathrm{obs}}) \mathbf{W} (A - A_{\mathrm{obs}})\, d\vec{x}\, dt$$

  where $\mathbf{W}$ is a matrix of statistical weights
- We want to minimize the cost function $\mathcal{J}$ given the constraint $\mathcal{E}(\overrightarrow{\Psi}, \vec{x}, t) = 0$.

The Lagrangian formulation

- The Lagrangian is :

$$\mathcal{L}(\overrightarrow{\Psi}, \overrightarrow{\lambda}) = \mathcal{J}(\overrightarrow{\Psi}) + \int_0^T \int_\Omega \overrightarrow{\lambda}(\vec{x}, t) \cdot \mathcal{E}(\overrightarrow{\Psi}, \vec{x}, t)\, d\vec{x}\, dt$$

  where $\overrightarrow{\lambda}(\vec{x}, t)$ are the Lagrange undetermined multiplicators also called adjoint variables (Sanders & Katopodes, 1999).
- Application of the variational operator δ to the Lagrangian :

$$\begin{aligned} \delta\mathcal{L} &= \overrightarrow{\nabla}_{\overrightarrow{\Psi}}\mathcal{L} \cdot \delta\overrightarrow{\Psi} + \overrightarrow{\nabla}_{\overrightarrow{\lambda}}\mathcal{L} \cdot \delta\overrightarrow{\lambda} \\ &= \frac{\partial\mathcal{L}}{\partial\overrightarrow{\Psi}}\delta\overrightarrow{\Psi} + \frac{\partial\mathcal{L}}{\partial\overrightarrow{\lambda}}\delta\overrightarrow{\lambda} \end{aligned}$$

- Minimum is reached only when $\delta\mathcal{L} = 0$

The Euler-Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial \overrightarrow{\lambda}} = \mathcal{E}(\overrightarrow{\Psi}, \vec{x}, t) = 0$$

and

$$\frac{\partial \mathcal{L}}{\partial \overrightarrow{\Psi}} = \text{Adj}(\overrightarrow{\lambda}) + \frac{\partial \mathcal{J}}{\partial \overrightarrow{\Psi}} = 0$$

where Adj$(\overrightarrow{\lambda})$ represents the adjoint equations (Schröter et al., 1993). This set of equations are the Euler-Lagrange equations.

Adjoint equation for the avalanche model

- Direct equation :

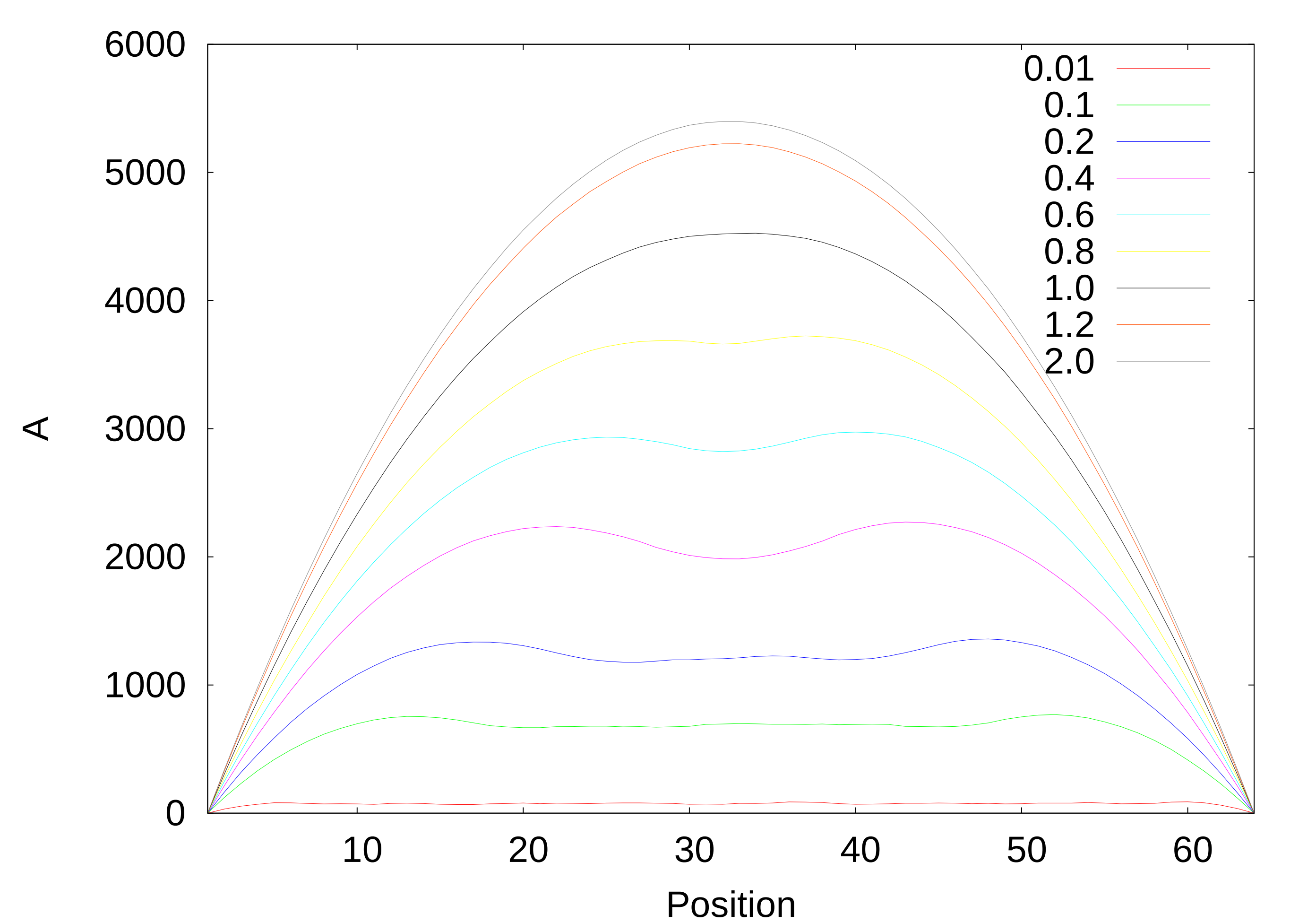
$$\frac{\partial A}{\partial t} = -\frac{\partial^2}{\partial x^2}\left(\nu(A_{xx}^2)\frac{\partial^2 A}{\partial x^2}\right) + F_R$$

- Adjoint equation :

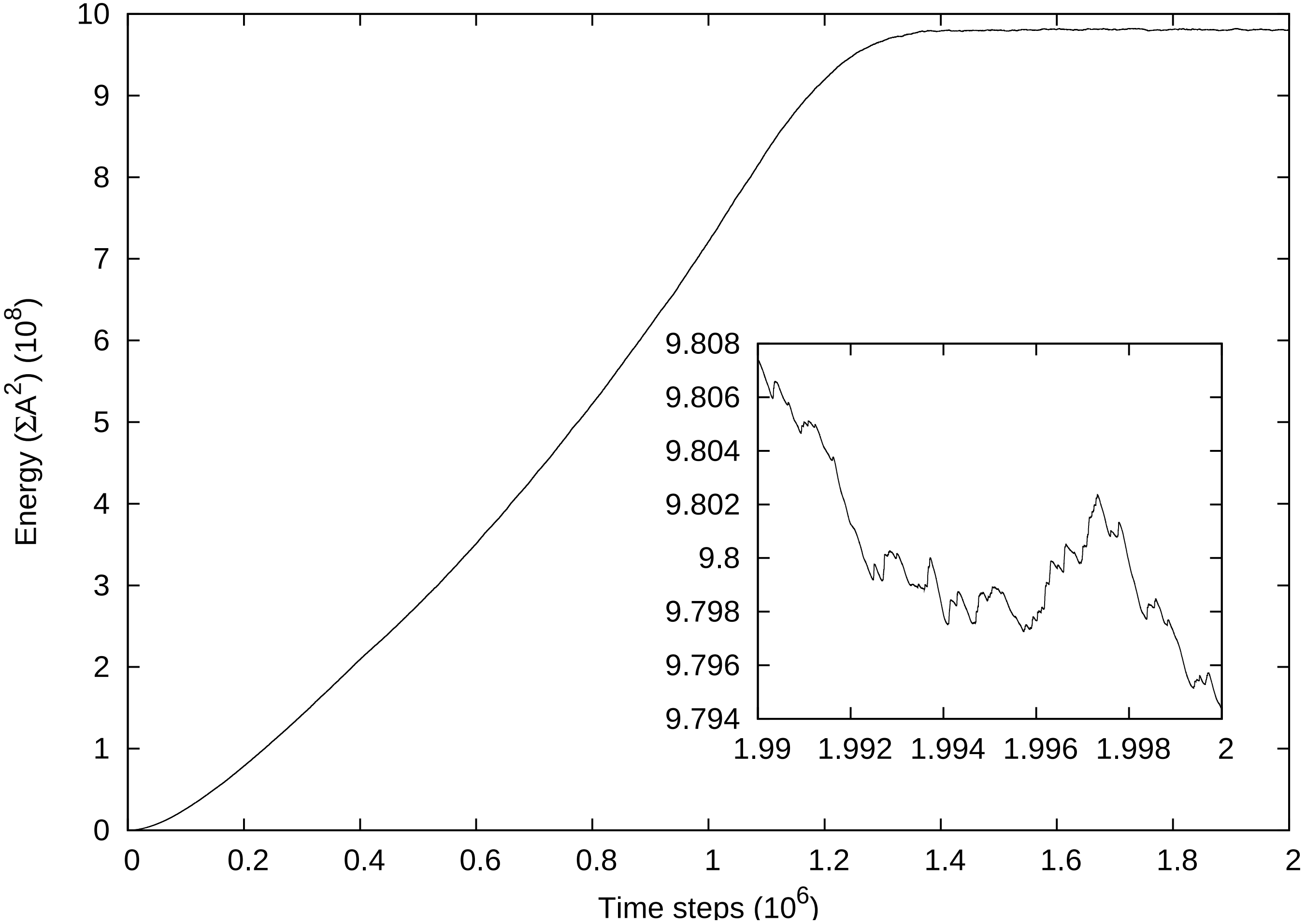
$$\frac{\partial A^*}{\partial \tau} = -\frac{\partial^2}{\partial x^2}\left(\nu(A_{xx}^{*2})\frac{\partial^2 A^*}{\partial x^2}\right) - \frac{\partial J}{\partial A}$$

Results

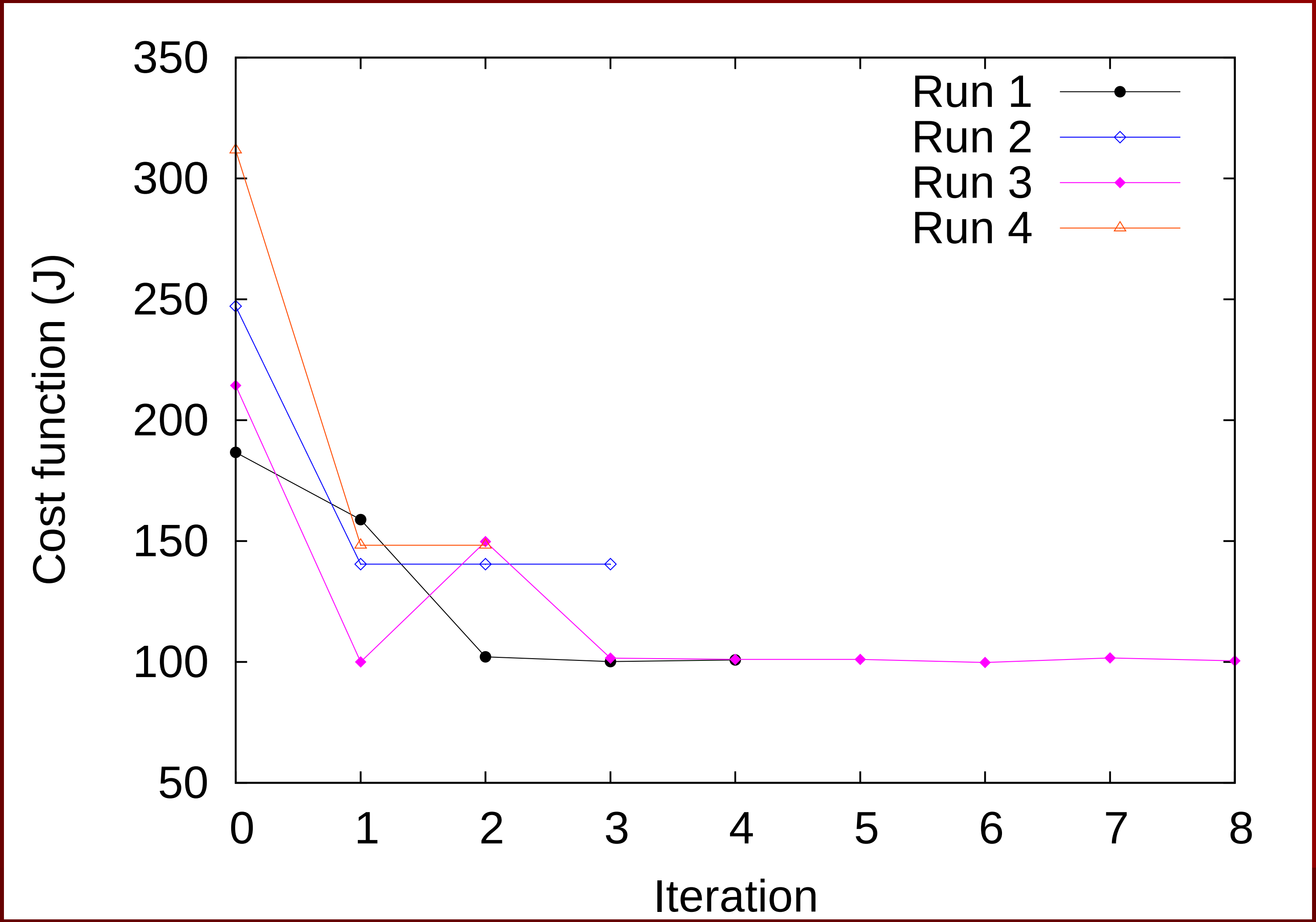
Toward the SOC state



Energy curve



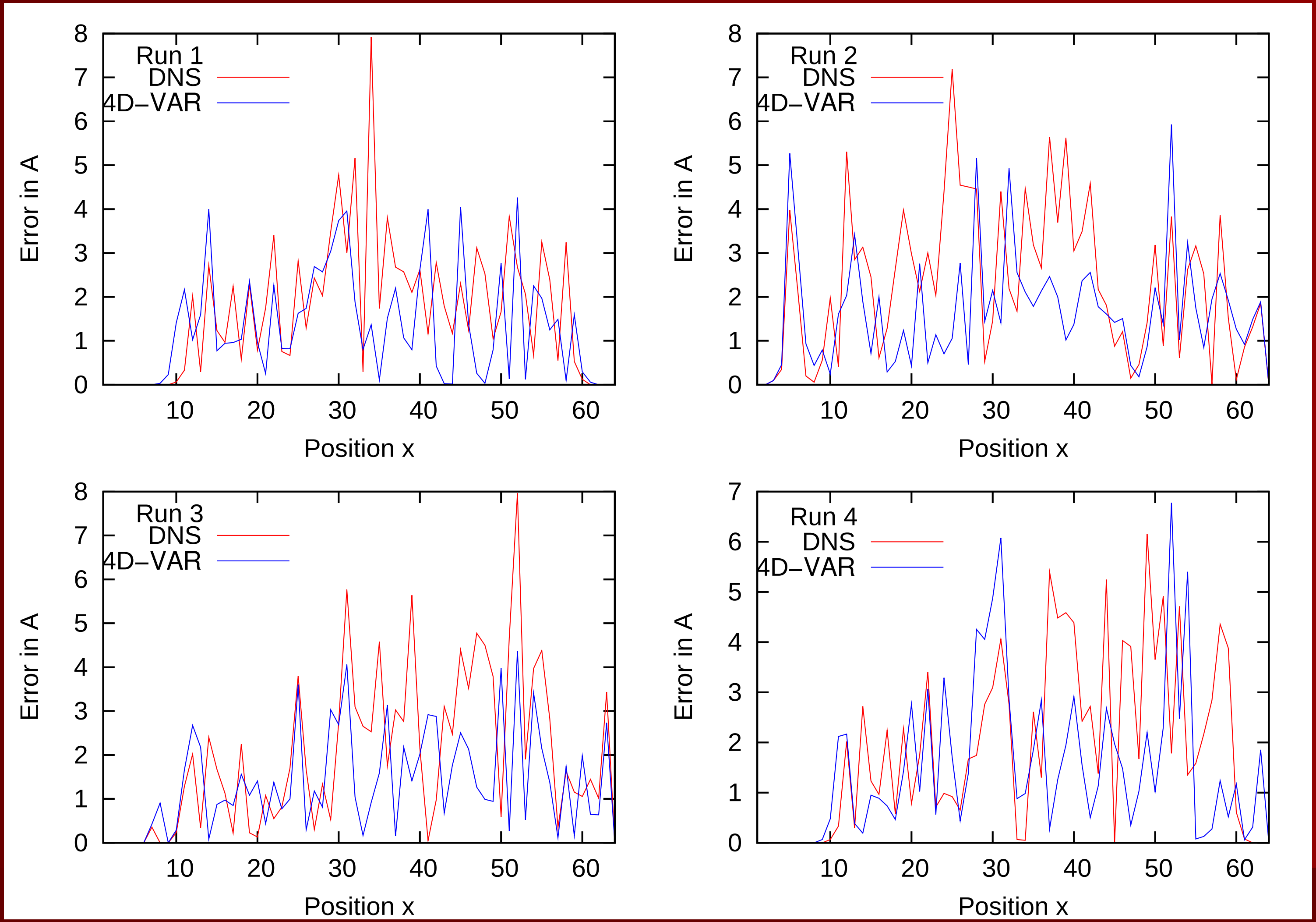
Minimisation of the cost function



DNS & 4D-VAR : mean error in A

Run	Mean DNS error	Mean 4D-VAR error
1	1.67	1.26
2	2.32	1.66
3	1.95	1.36
4	1.88	1.49

DNS & 4D-VAR : error in A



DNS & 4D-VAR : energy