L’utilisation des modèles d’avalanche 2D et de l’assimilation de données dans l’étude des éruptions solaires

The Use of 2D Avalanche Models and Data Assimilation in the Study of Solar Flares

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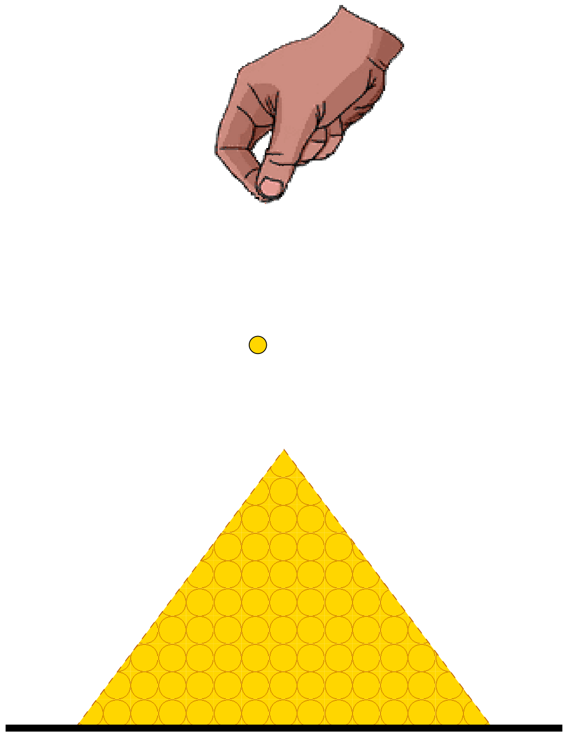
Summary

- Self-organized critical (SOC) models
- Data assimilation methods
- Example : solar flares
  - Quick introduction to solar flares
  - Application of SOC and data assimilation to solar flares

SOC Models

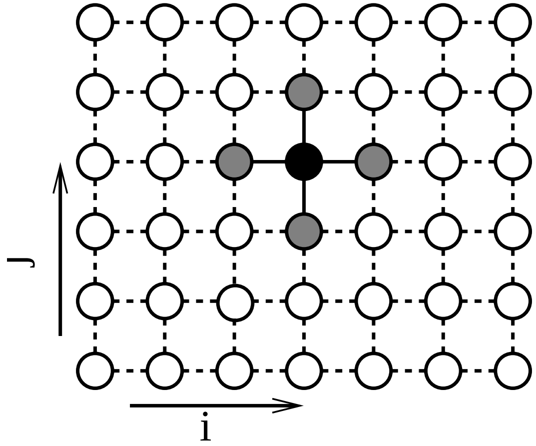
Self-organized criticality



- an open system with slow external forcing
- a self-stabilizing instability threshold
- a local redistribution of a dynamic variable

Avalanche model

- Lattice of metastable sites (Charbonneau et al., 2001)



- Stability criterion :

$$\Delta A^n_{i,j} \equiv A^n_{i,j} - \frac{1}{2D} \sum_{\text{neighbours}} A^n_{\text{neighbours}}$$

- Redistribution of A :

$$A^{n+1}_{i,j} = A^n_{i,j} - \frac{2D}{2D+1} \Delta A^n_{i,j}$$
$$A^{n+1}_{i\pm1,j\pm1} = A^n_{i\pm1,j\pm1} + \frac{1}{2D+1} \Delta A^n_{i,j}$$

Avalanche model (continued)

- Continuous avalanche model :

$$\frac{\partial A}{\partial t} = -\frac{\partial^2}{\partial x^2}\left(\nu(A_{xx}^2)\frac{\partial^2 A}{\partial x^2}\right) - \frac{\partial^2}{\partial y^2}\left(\nu(A_{yy}^2)\frac{\partial^2 A}{\partial y^2}\right) + F_R$$

with :

$$\begin{matrix} \nu(A_{xx}^2) \\ \nu(A_{yy}^2) \end{matrix} = \begin{cases} \nu_a & \text{if } \Delta A^2 > A_c^2 \\ 0 & \text{else} \end{cases}$$

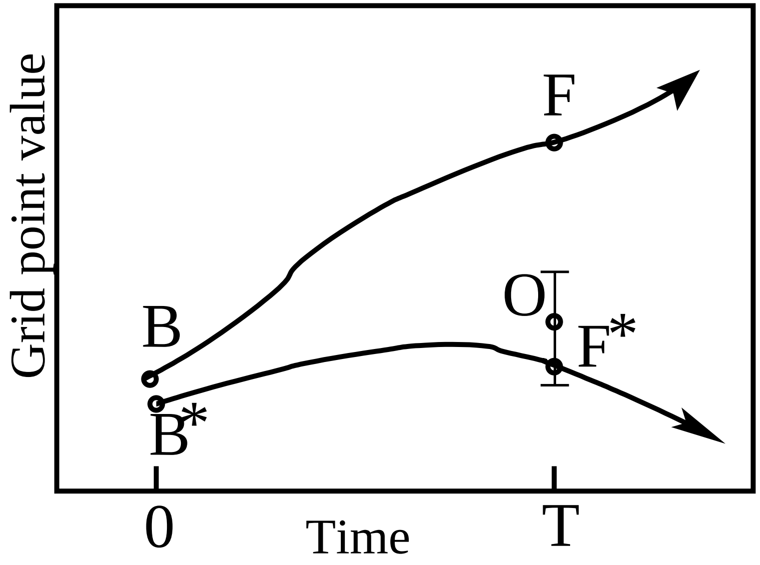
and F_R is a random forcing term.

Data Assimilation Method

Data Assimilation

Data assimilation is an efficient technique to incorporate observations in numerical models in order to make forecasts.

Variational (4D-VAR) methods



- B : estimation of the system initial state
- F : forecast at time T
- O : observation
- B^* : correction of the initial state estimation
- F^* : new forecast at time T

Lyapunov exponents

- In a time interval Δt, a small initial perturbation (error) $\epsilon(\Delta t)$ grows exponentially :

$$\epsilon(\Delta t) \propto e^{\Lambda \Delta t}$$

  where Λ is a Lyapunov exponent.
- $\Lambda \leq 0 \Longrightarrow$ stable system
- $\Lambda > 0 \Longrightarrow$ unstable system

The cost function

- Generally, the cost function is written as :

$$\mathcal{J} = \int_0^T \int_\Omega f(\overrightarrow{\Psi}, \vec{x}, t)\; d\vec{x}\; dt$$

- More precisely, for data assimilation, :

$$\mathcal{J} = \frac{1}{2} \int_0^T \int_\Omega (A - A_{\text{obs}}) \mathbf{W} (A - A_{\text{obs}})\; d\vec{x}\; dt$$

  where $\mathbf{W}$ is a matrix of statistical weights
- We want to minimize the cost function $\mathcal{J}$ given the constraint $\mathcal{E}(\overrightarrow{\Psi}, \vec{x}, t) = 0$.

The Lagrangian formulation

- The Lagrangian is :

$$\mathcal{L}(\overrightarrow{\Psi}, \overrightarrow{\lambda}) = \mathcal{J}(\overrightarrow{\Psi}) + \int_0^T \int_\Omega \overrightarrow{\lambda}(\vec{x}, t) \cdot \mathcal{E}(\overrightarrow{\Psi}, \vec{x}, t) \, d\vec{x} \, dt$$

  where $\overrightarrow{\lambda}(\vec{x}, t)$ are the Lagrange undetermined multiplicators also called adjoint variables (Sanders & Katopodes, 1999).
- Application of the variational operator δ to the Lagrangian :

$$\begin{aligned} \delta\mathcal{L} &= \overrightarrow{\nabla}_{\overrightarrow{\Psi}}\mathcal{L} \cdot \delta\overrightarrow{\Psi} + \overrightarrow{\nabla}_{\overrightarrow{\lambda}}\mathcal{L} \cdot \delta\overrightarrow{\lambda} \\ &= \frac{\partial\mathcal{L}}{\partial\overrightarrow{\Psi}}\delta\overrightarrow{\Psi} + \frac{\partial\mathcal{L}}{\partial\overrightarrow{\lambda}}\delta\overrightarrow{\lambda} \end{aligned}$$

- Minimum is reached only when $\delta\mathcal{L} = 0$

The Euler-Lagrange equations

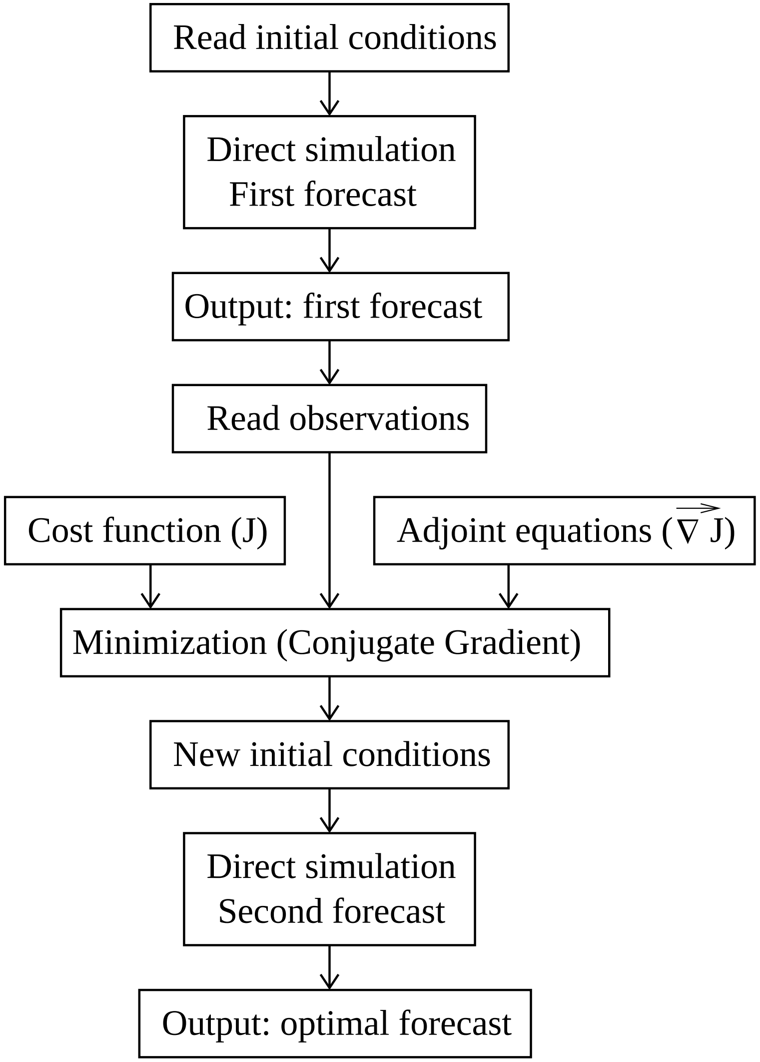
$$\frac{\partial \mathcal{L}}{\partial \overrightarrow{\lambda}} = \mathcal{E}(\overrightarrow{\Psi}, \vec{x}, t) = 0$$

and

$$\frac{\partial \mathcal{L}}{\partial \overrightarrow{\Psi}} = \text{Adj}(\overrightarrow{\lambda}) + \frac{\partial \mathcal{J}}{\partial \overrightarrow{\Psi}} = 0$$

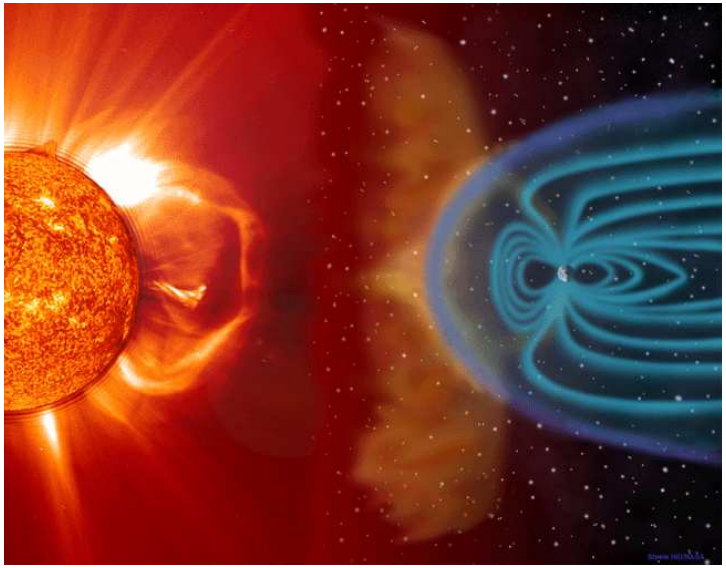
where Adj$(\overrightarrow{\lambda})$ represents the adjoint equations (Schröter et al., 1993). This set of equations are the Euler-Lagrange equations.

Algorithm of the 4D-VAR method



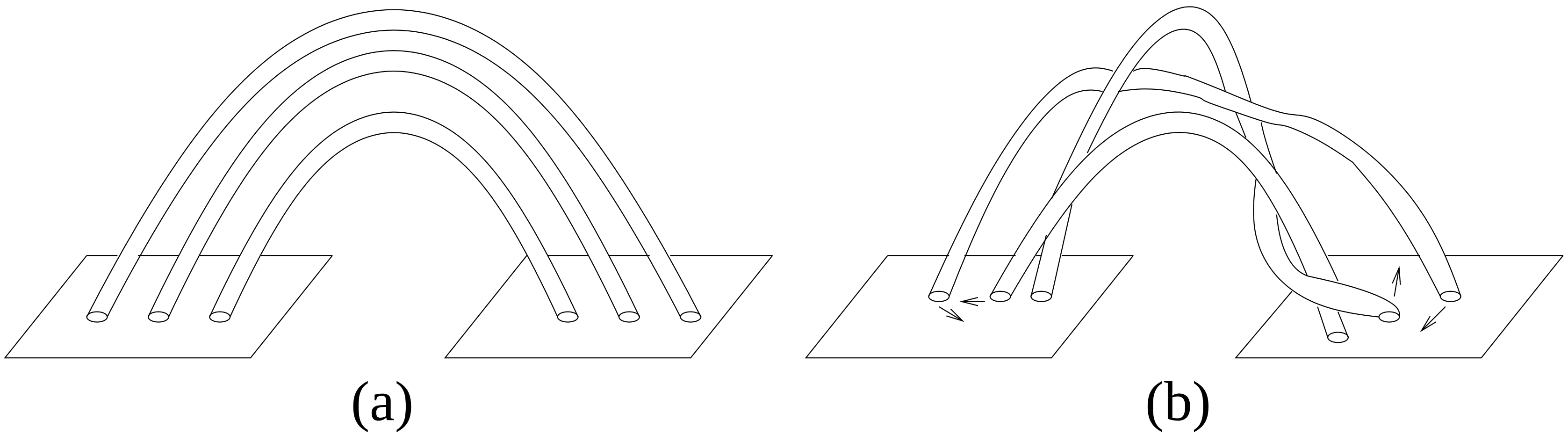
Example : Solar Flares

The Sun-Earth connection



- Magnetosphere and solar wind are in an equilibrium state
- Solar flares may cause coronal mass ejection (CME)
- Perturbed equilibrium : $\frac{d\vec{B}}{dt} \neq 0$
- Faraday's Law : $\frac{d\vec{B}}{dt} = -\vec{\nabla} \times \vec{E}$
- Electrical currents flowing in the ionosphere and in the ground

Flux tubes twisting (Parker, 1983)

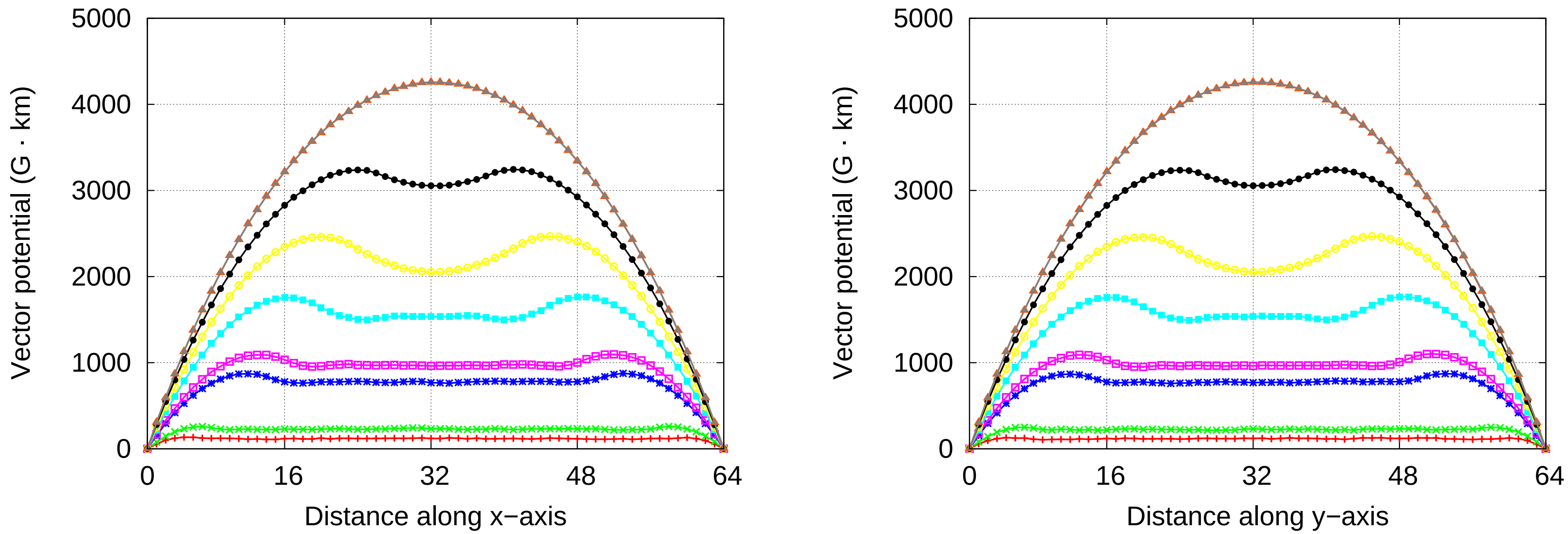


- Fig. (a) : A magnetic loop emerging from the Sun.
- Fig. (b) : At the photosphere, the flux tubes are twisted and intertwined by the random motion of the convective cells. Solar flares are the liberation of energy coming from the magnetic reconnection of such tangential discontinuities.

Power law (Aschwanden et al., 2000)

- Several observed characteristics of solar flares behave as power laws :
  - Released energy (up to 10 orders of magnitude)
  - Waiting time distribution between flares
  - X-ray flux distribution maximum
- These numerous power laws indicates a scale invariance where all flares are governed by the same laws of physics.
- SOC models can reproduce these power laws without any adjustable parameter.

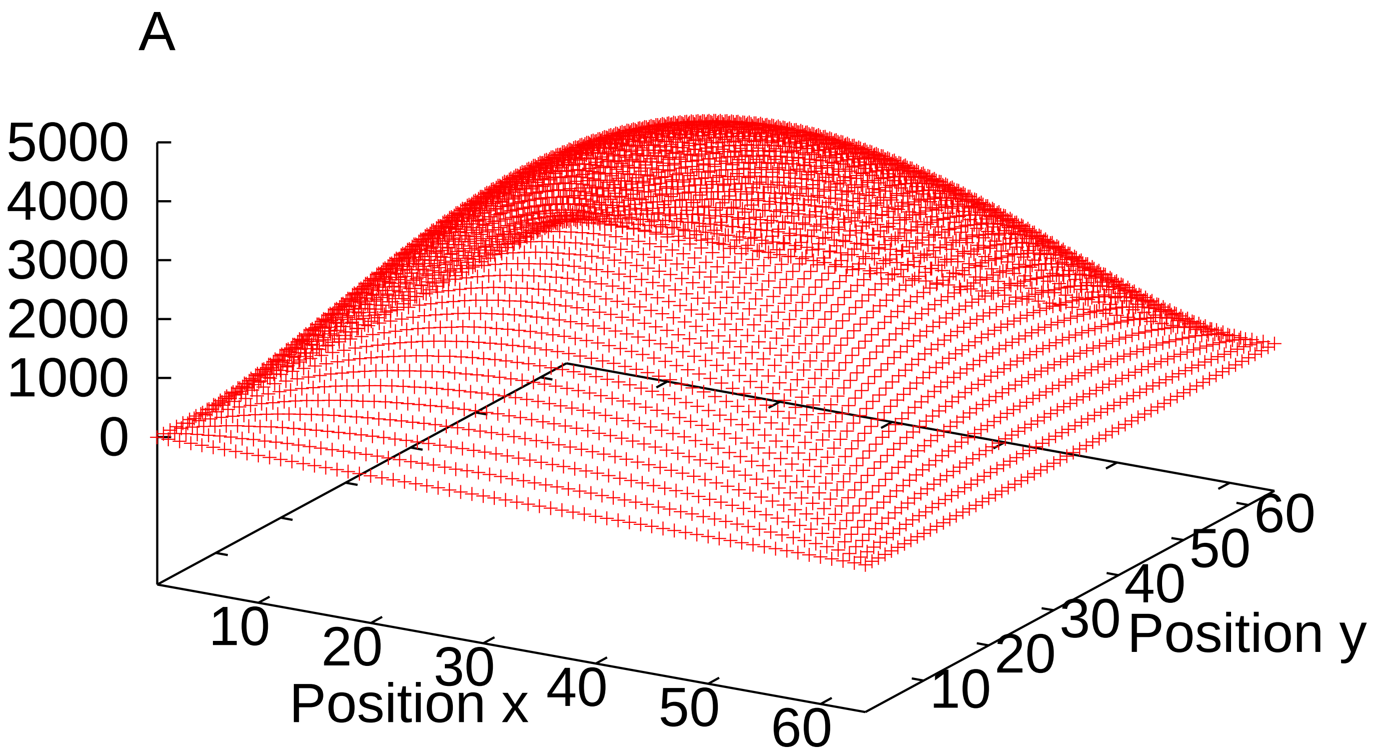
Toward the SOC state



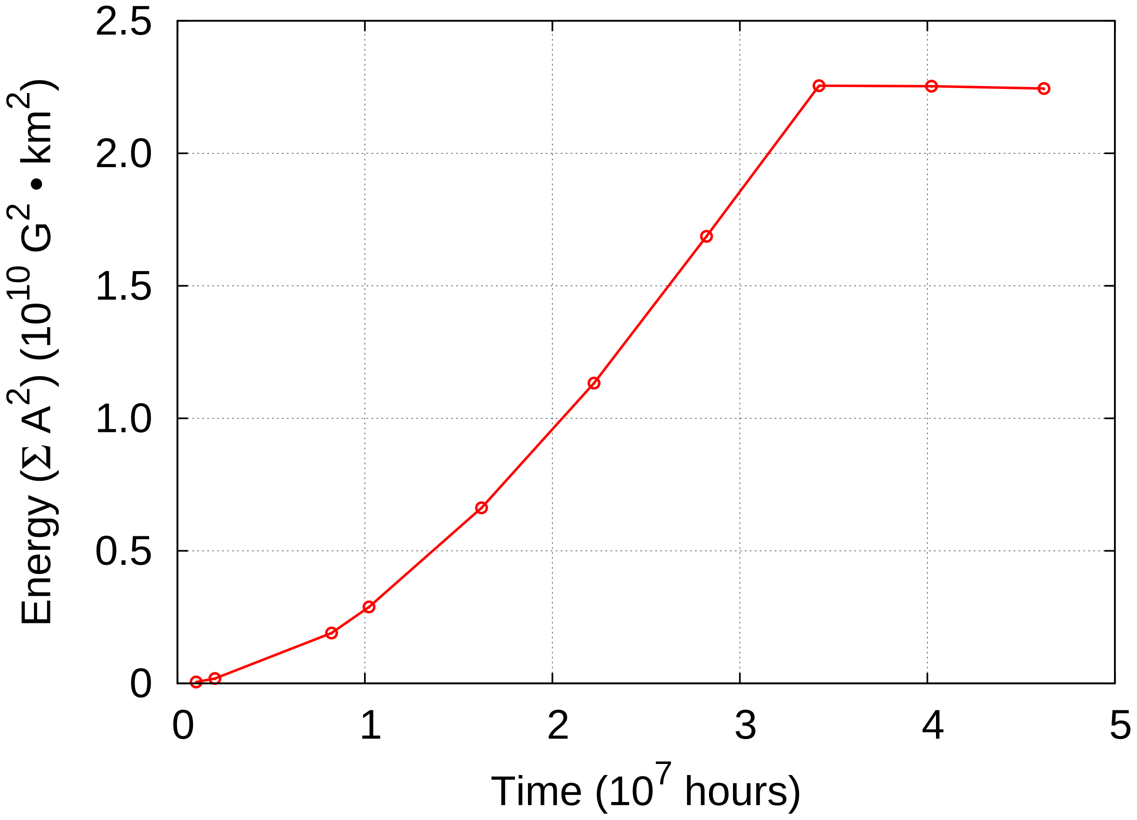
(a) Cross-section along the y-axis (b) Cross-section along the x-axis

Evolution of the system from a null initial state ($A = 0$ everywhere) to the SOC state (the inverse parabola). There is a clear symmetry between the x and y direction.

Contour plot of the domain



Energy curve



- Increase of energy ($\sum A^2$) with time.
- The plateau at $t \sim 3.5 \times 10^7$ means that SOC as been reached.

Adjoint equation for the avalanche model

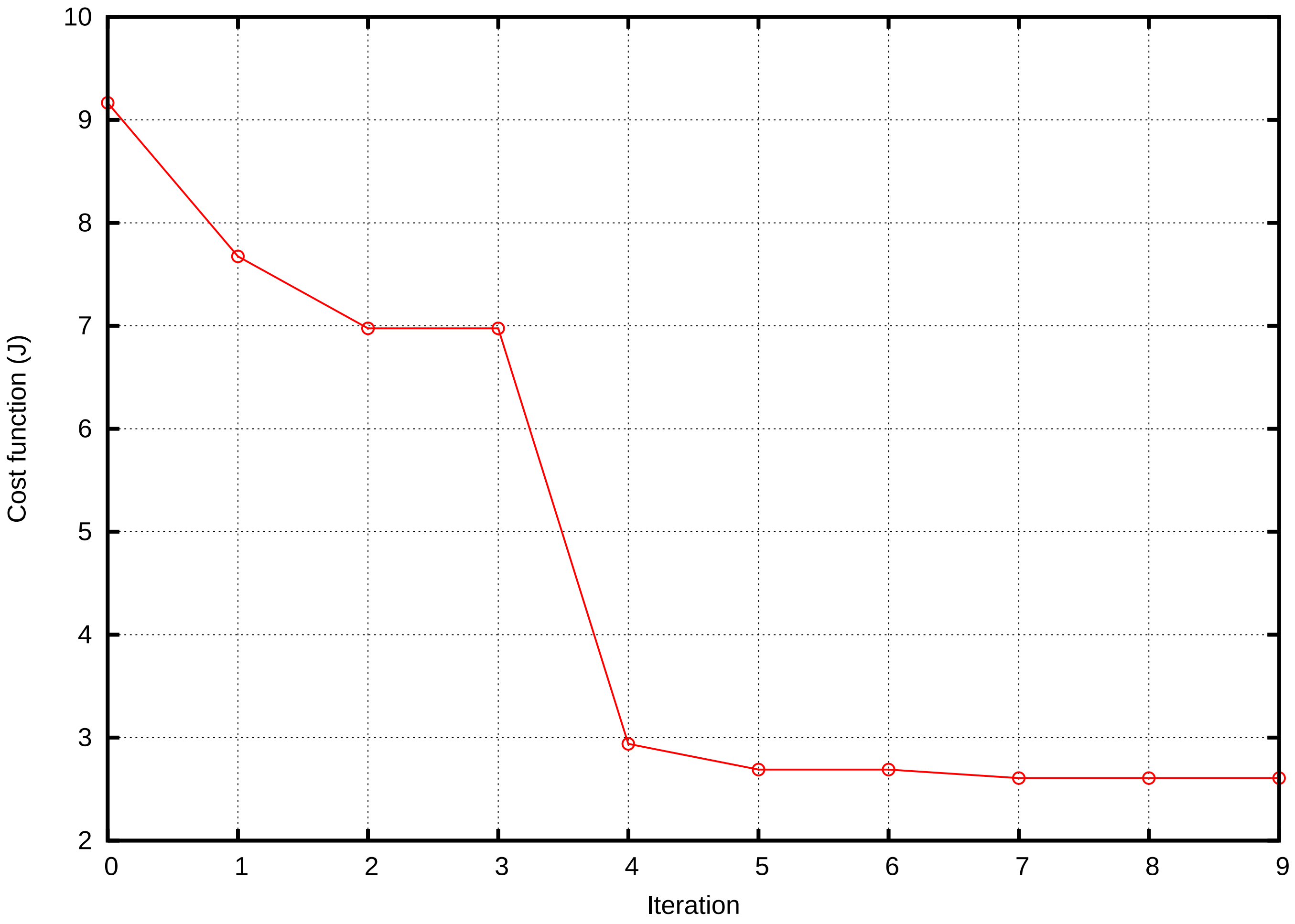
- Direct equation :

$$\frac{\partial A}{\partial t} = -\frac{\partial^2}{\partial x^2}\left(\nu(A_{xx}^2)\frac{\partial^2 A}{\partial x^2}\right) - \frac{\partial^2}{\partial y^2}\left(\nu(A_{yy}^2)\frac{\partial^2 A}{\partial y^2}\right) + F_R$$

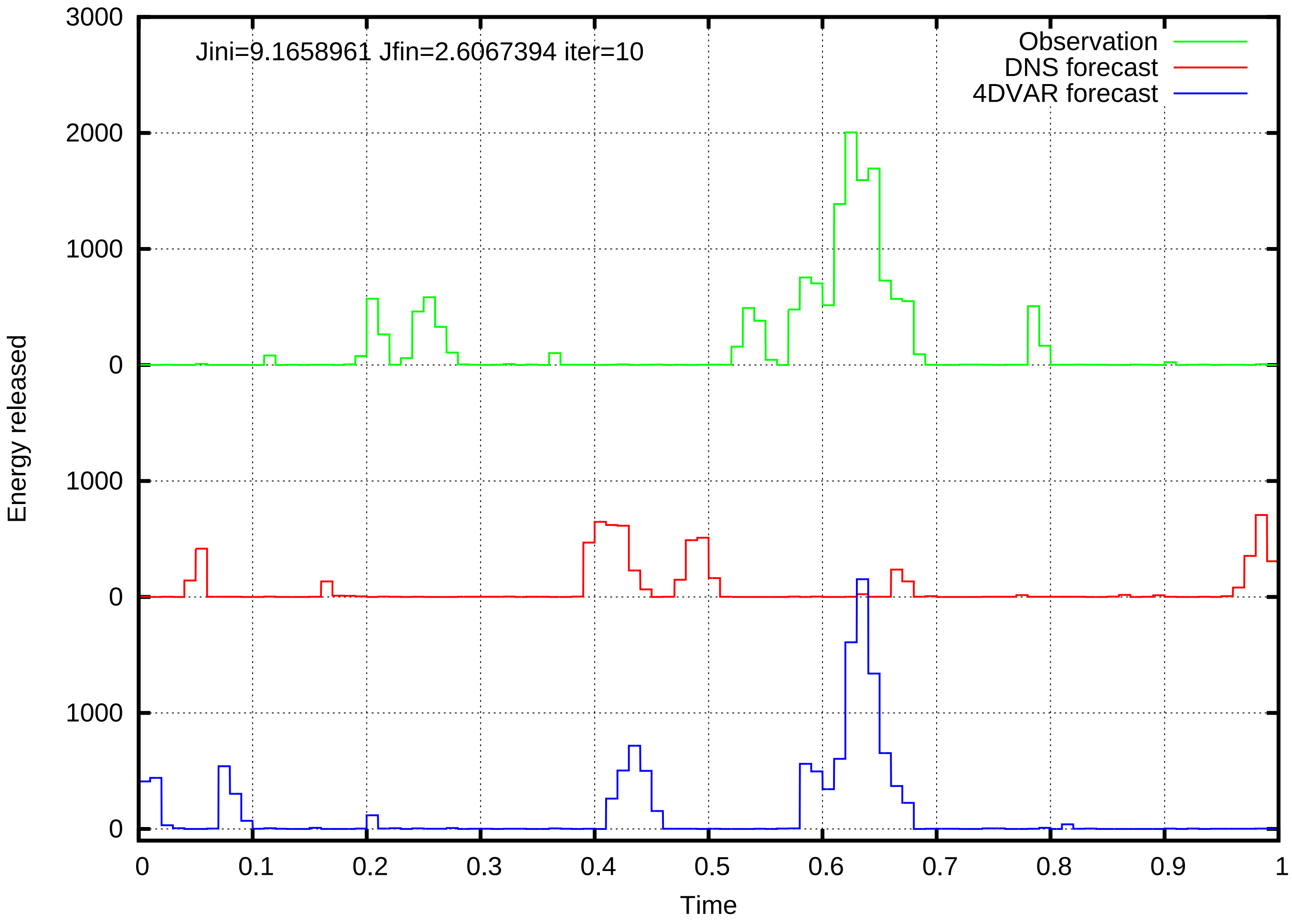
- Adjoint equation :

$$\frac{\partial A^*}{\partial \tau} = -\frac{\partial^2}{\partial x^2}\left(\nu(A_{xx}^{*2})\frac{\partial^2 A^*}{\partial x^2}\right) - \frac{\partial^2}{\partial y^2}\left(\nu(A_{yy}^{*2})\frac{\partial^2 A^*}{\partial y^2}\right) - \frac{\partial J}{\partial A}$$

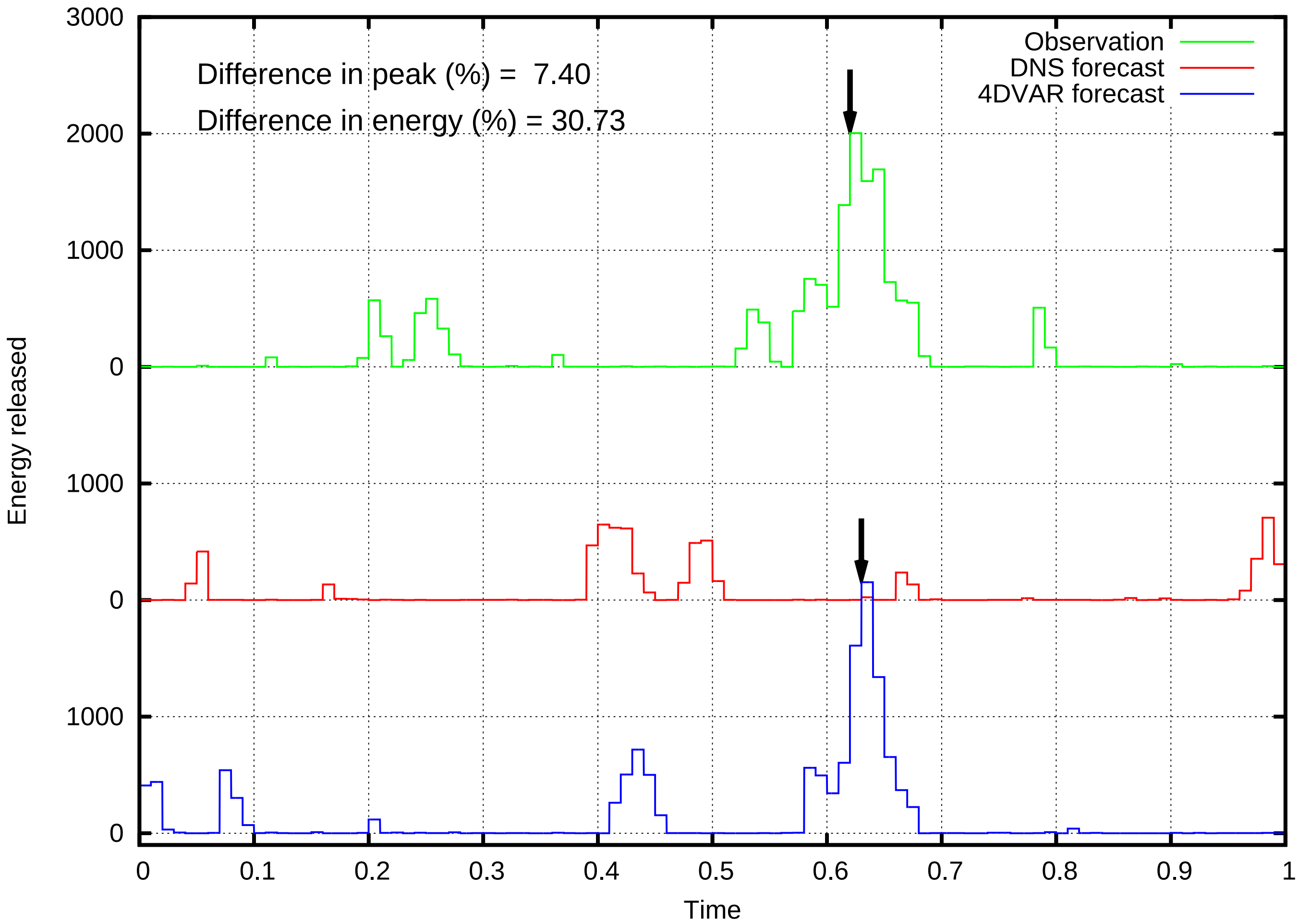
Minimization of the cost function



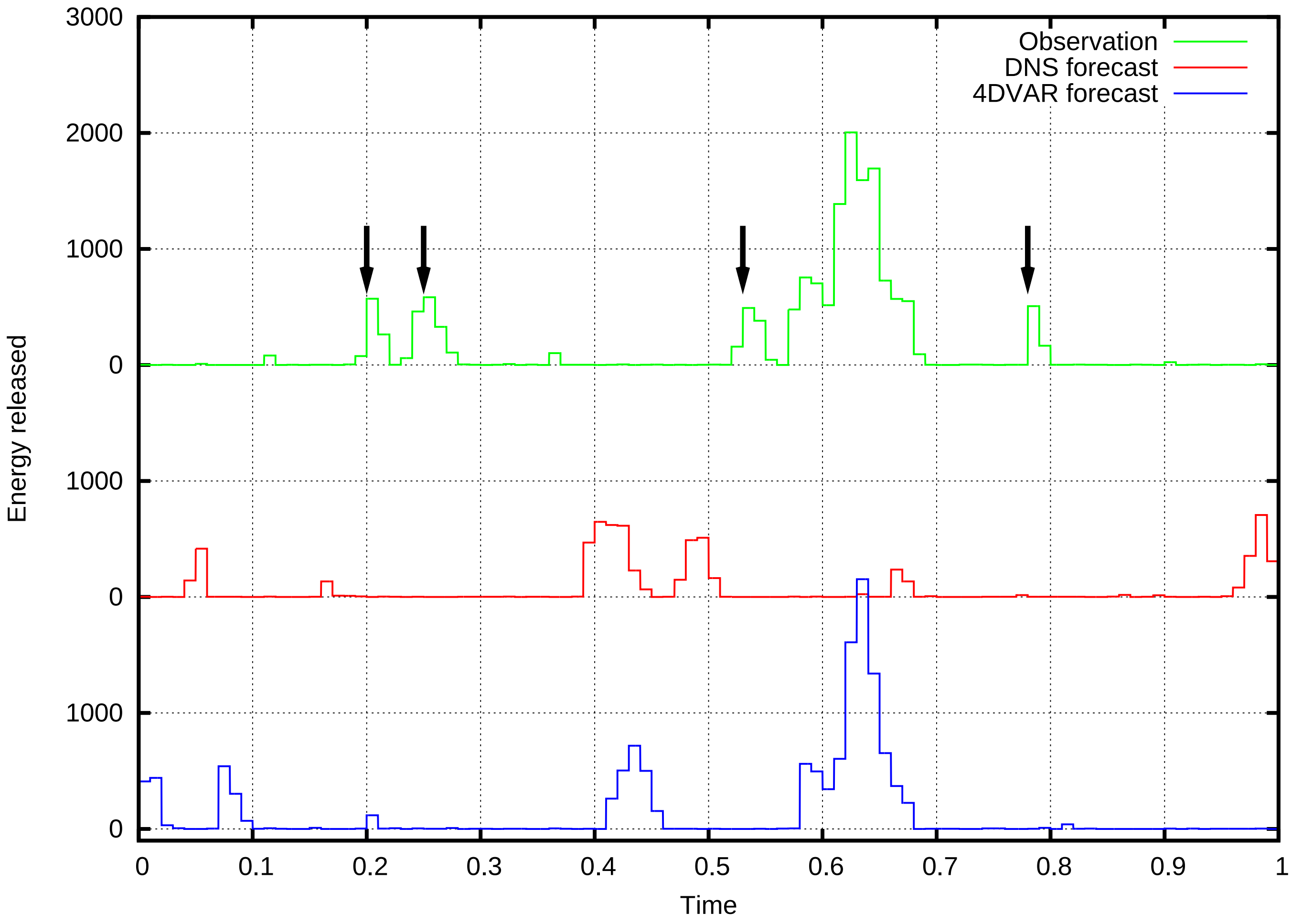
DNS vs. 4D-VAR : Comparison



DNS vs. 4D-VAR : Match



DNS vs. 4D-VAR : Misses



DNS vs. 4D-VAR : False alarms